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Methodical Specifics of Solving Some Mathematical Olympiad Problems

The Author's Abstract

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General description of paper

Topicality of research. Mathematics, as a school discipline, is a key subject in secondary school in many aspects: it facilitates logical thinking, the development of critical thought and, helps pupils in the formation of a spatial representation and is a key discipline for completing natural science subjects, since as in most cases, studies in subjects of natural science cycle are carried out using mathematical methods. It should be noted that the use of mathematical methods in subjects of natural science cycle very often happens by demonstrating particular mathematical problems, which increases the interest of pupils in studying mathematical problems, and, to some extent, expands the topics of problems to be solved in mathematics curricula.

The specificities of teaching the initial course of mathematics are that this phase mostly emphasizes solving the problems, and studies of the theoretical material are minimized. This approach provides pupils with a positive attitude towards problem solving, and it strengthens their belief that mathematics should be studied through problems. The foundations of such an approach are undermined by the introduction of teaching a systematic course of the material with geometric content in the mathematics course, and not only by studying the mathematical material with that content. The process of problem solving at the second stage of public schools is based on knowledge acquired from the theoretical teaching content of the curricula, and there is really formed an indicator of the ratio of time intended for teaching the theoretical issues, and time intended for solving problems, which is expressed by the excess of the time allotted for acquiring theoretical knowledge not only on the second level, but also on the third level.

We believe at the secondary and third levels of school education that mathematical problems cannot be solved without theoretical knowledge, but we think that the duration of time intended for solving mathematical problems should be at least twice the time of teaching the theoretical material. In other words, in teaching mathematics, our preference is for teaching through problem solving, which obviously does not mean that pupils are completely divorced from theoretical knowledge.

It is our view that transfer of theoretical knowledge to pupils in math class should be carried out by the same methods and forms as had occurred until now, but a special emphasis should not be placed on the strict approaches of proving mathematical theorems and provisions, as well as on the details of the proving process. The issues should be explained on the basis of ascertaining the mathematical facts and provisions, as a result of which, the process of knowledge strengthening should be carried out by means of problems selected specifically for the training subject. Sometimes, we believe it would be expedient to transfer to pupils proving the particular theorem or provision of the mathematics school course in a form of problem that raises the level of knowledge of pupils and enhances mathematics application capabilities in terms of both intradisciplinary and interdisciplinary relationships.

Studies carried out by international organizations have shown that schoolchildren from the age of 12-13 before graduating from secondary school (this age corresponds

to the second and third levels of Georgian educational space) have a better understanding of mathematical theoretical issues rather than methods of solving the problems that have been already passed and studied. One of the topical issues of mathematics teaching methodology is to find that middle ground, which should be established as the correlation between theoretical material and solving mathematical problems. The authors of the math textbooks recommended by the Ministry of Education for the schools of Georgia do not say anything about what should be this correlation, but it is practically confirmed that the balance is weighted in favor of theoretical teaching at the second and third levels of school. We do not agree with such an approach and believe that we need to spend at least twice the number of hours on solving the problems compared to learning theoretical material. We believe that theoretical issues should be included in the teaching process in the form of problems, and we should avoid close attention to details of theorems and mathematical provisions with strict mathematical judgements.

The mathematical school course teaching should be carried out through the specifically chosen mathematical problems only when solving problems. It should also be taken into consideration that the problem cannot be solved only by solving large number of problems, of particular importance are the content of the problems and methods of solving them.

In recent years, Georgian mathematical Methodists G. Berdzulishvili, G. Bregadze, N. Oniani-Saghinadze, B. Bakuradze, I. Gogiberidze, T. Tsereteli and others believe that teaching special methods of solving mathematical problems at all three levels of school plays an important role in acquiring mathematical knowledge by pupils and in their mental and personal development. They have developed special methodologies for teaching solving and composing problems solvable by special methods and containing specific mathematical topics by secondary school levels, the effectiveness of which by including them in teaching process is confirmed by the appropriate pedagogical experiments.

To develop skills of problem solving, the methodology for mathematics teaching considers two different approaches: pupils are required to solve as many problems as possible, or they have to master the special methods and/or techniques for solving mathematical problems, where the content of problem is important as well.

It is unacceptable for us to teach pupils solving problems without solving them in practice, but we believe that when teaching mathematics through mathematical problems, of high importance is the content of mathematical problems, as well as the appropriate method or/and technique for solving them that will enable pupils to develop skills appropriate with the topic of problem, and which should be required for developmental teaching.

We believe that not only how well is understood the knowledge of methods and techniques for solving problems by pupils, but also how deeply, thoroughly and firmly is acquired this knowledge by pupil is one of the most important objectives of teaching, and simultaneously, is a constituent part of a general methodological didactic principle of teaching. This principle implies, first of all, such transfer of theoretical

material that allows pupil for memorizing, and on that basis, for being able to remember freely what he/she had previously acquired and learnt in teaching process.

We also believe that teaching mathematics within the school course through the specifically selected problems should be carried out at all three educational levels, but since proceeding from the specificity of teaching at the primary educational level, this issue has already been partially resolved, so the main focus in our dissertation has been shifted to the second and third educational levels, where the mathematics teaching should be carried out by including the school mathematical olympiad problems in the teaching process, and the methodological specifics for solving them should be developed. It is obvious that within the framework of one dissertation, it is impossible to discuss the olympiad problems for all types of the school mathematics courses, which can be incorporated into mathematical school curriculum, and this topic is inexhaustible and requires further research. We shall confine ourselves to the topics of the mathematics school course, such as: the methodical specificities of teaching some special techniques for solving trigonometric equations and inequalities of olympiad content; solving some types of the mathematical olympiad problems using the interdisciplinary relationships; the methodical specificities of teaching some non-standard techniques for solving some types of the mathematical olympiad problems in a set of integers; solving some types of the mathematical olympiad algebraic problems through the geometric interpretation; the methodical specificities of teaching some special techniques for solving the functions-related mathematical olympiad problems, etc. The dissertation has Annex containing more than two hundred mathematical problems with a school olympiad content, which we recommend teachers to include them in teaching process or to give to pupils as homework assignment, which will help them in receiving a broad-based and in-depth mathematical, and this does not require to allocate additional time. Proceeding from teaching process, mathematics teacher can compose independently problems similar to those, which are shown in Annex in accordance with different topics, which will be included in teaching process and/or will be used for giving homework assignment. Consideration of such problems during the teaching process greatly effective during the summarizing repeated class, when establishing interrelations between the taught and new materials, and so on.

Strengthening pupils' knowledge and skills is closely linked to the relationship between the studied and new materials, and for this, it is necessary to observe the principles of didactics, particularly, a teacher should not start teaching the methods and techniques of solving the mathematical olympiad problems until pupils study thoroughly the algorithmic methods of problem solving, and at the same time, he/she is required to comply with the didactic requirements of transition from studied to new material, the most important of which is to reduce the algorithmic techniques of problem solving to a system, and to separate the primary from the secondary aspects, to set the problem and to outline the new issue plot in a manner that enables pupil to clearly see the need for teaching special techniques for solving the mathematical olympiad problems. All this will allow a teacher for motivating pupils to study special

methods and techniques of solving the mathematical olympiad problems, and for getting them appropriately psyched up to easily cope with them.

For mastering special techniques for solving the mathematical olympiad problems effectively and in a short period of time, great importance is attached to the systematic mathematical practice, which should be correlated with the complexity and simplicity of the problems for solving them by special techniques and methods, and with their specificity.

The process of teaching the school mathematical olympiad problem solving should be built by teacher on their content and logical understanding, which is based on the active memorization and logical thinking, because the full development of the human person from pupils requires, along with a number of other measures, require such reaching, which enable pupils to learn consistently the foundations of the sciences.

Secondary school mathematics curricula determine which topics should be studied by pupil in each class, that is, the types classes of the algorithmitaclly solvable exercises are defined. Obviously, it is impossible to achieve a high level of mathematical training of pupils only by teaching the algorithmitaclly solvable problems.

The State educational standard for mathematics does not restrict teachers from choosing the algorithmic and special methods of teaching problem solving. The standard does not make explicit the content of the mathematical school course, and it just contains the issues that must be included in the course (in the form of introduction or a systematic course). Therefore, the standard requirements can be met by different versions of programs and textbooks.

The secondary school mathematics curricula and the State educational standards for mathematics do not clearly formulate the requirements that pupil must have after the completion of the mathematics course. They indicate that pupil is required to have not only the mathematical knowledge and skills that are achieved through the algorithmitaclly solvable problems, but he/she is also required to be able to solve the problems, which are reduced to the algorithmic problems. In other words, through the use of special techniques for problem solving, the pupils are expected to be able to reduce the algorithmitaclly solvable olympiad problems such kind of problems, whose solution is known to pupils, and it is algorithmic.

The process of forming pupils' creative thinking is subject to the strong influence of those problems, which are not directly solved by the known algorithms. The process of solving such problems requires cognitive thinking, which enhances the joy of finding the way of solving the problem, and this is an emotional factor and is a powerful leverage over pupil's behavior. Proper management of the emotional factor is of utmost importance in all areas of human activity, as well as in the process of forming and teaching of pupil as a complete person.

Emotions and spiritual experiences of pupils, as exponents of their subjective state, are mostly associated with the teaching process, so giving the right direction to emotions and simultaneously, using them as psychological factor to cope with

problem, is the exclusive prerogative of the school and teacher. Qualitative transformation and development of pupils' emotions is mostly carried out through upbringing and teaching.

Emotions caused by solving the mathematical school olympiad problems should be used by teacher as an integral part of the educational process. Therefore, teachers should take into consideration the specific ways and measures to ensure positive emotions and spiritual experiences in pupils, and, at the same time, to make fun and attractive the training process. Teachers should also rejoice with pupils and should help pupils to overcome all obstacles and difficulties, and they should also be familiar with their aspirations, emotions, observe and study the development and transformation of emotions during the educational process, as well as they should carry out the development of pupils in the right direction, using an individual approach to every one of these pupils.

Analysis of problems to be solved in the secondary school mathematics courses reveals that too little time is spent on those problem in this course, whose solution requires the use of methods and techniques, which is why we believe that the problems given in the existing secondary-school textbooks for the second and third level mathematics have a lesser developmental effect and does not meet the modern requirements. This trend is not new, it has been so in the past, and it is so today. Indeed, if we look closely to the past, we will see that it was generally recognized by specialists in mathematics teaching methodology that the systems of problems existing in the secondary-school textbooks for mathematics have a number of shortcomings. One of the major shortcomings we've emphasized above is the detachment of the theory from practice. A.Stolyar remarked that math classes are basically focused on a training solution of the abstract problems formulated in mathematical terms; I.Kolyagin, V Oganesian and others believe that the school math exercises are scarce in terms of their cognitive nature and content. I. Guidionov points to a lack of diversity among the types of the mathematical school course exercises, A.Dograshvili believes that the school textbooks contain a small number of probabilistic and combinatorial problems, T.Moralishvili believed that, the exercises system in the mathematics school course is not structurally complete, T.Tsereteli and G.Berdzulishvili believe that the school course does not contain such geometrical problems, whose solving requires the use of special methods. G.Berdzulishvili believes that the teaching process should involve problems with the interdisciplinary and intrasubject relationships, N. Oniani-Saghinadze and B. Bakuradze think that in the elementary and secondary school, less attention is paid to the system of problems with the content of the sets and probability theory, N. Nakhutsrishvili believes that, at the elementary school level, it is necessary to give center stage to the problematic and developmental matters, G.Nozadze and M.Ochkhikidze believe that the school textbooks for mathematics lack the developmental problems, I. Chkhikvadze believes that, for developing teaching in the elementary classes, it is necessary to use more story problems, and so on.

The types of the training exercises given in the secondary-school textbook for mathematics, with their functional purposes, correspond to the traditional scheme

„theory → problems ". We tend to have an assumption developed in recent period that teaching should be carried out in accordance with a scheme „problems → theory → problems ". Such an approach is essential in the secondary school teaching practice for the activation of the principles of the developmental teaching, when teaching is mostly carried out through the problem solving. The scheme „problems → theory → problems "allows us for transferring to pupils and strengthening not only knowledge of theoretical material, but also for helping them to acquire new knowledge independently through their use.

The modified scheme of teaching requires a suitable exercise system, which also contains the functional-purpose exercises different from the existing ones. To that end, there has been analyzed the experience not only of Georgian and post-Soviet space but also European experience.

More recently, Georgian experts in mathematics teaching methodology G.Berdzulishvili, G.Bregadze, N. Nakhutsrishvili, B. Bakuradze, L.Kurchishvili, G. Jinjikhadze, L. Babunashvili, N. Oniani-Saginadze, L. Tsibadze, I.Gogiberidze and others have systematically developed and improved the systems of problems, which are based on the scheme „problems → theory → problems", G.Berdzulishvili, G.Bregadze, N.Nakhutsrihrishvili, B. Bakuradze, L.Kurchishvili, G.Jinjikhadze, L.Babunashvili, N. L. Tsibadze, I.Gogiberidze, and others.

Inclusion of the mathematical olympiad problem solving in the secondary school mathematics course should be carried out using the scheme „problems → theory → problems", which requires to build innovatively a system of problems to be considered in during the classes.

All above stated allows us for giving a single significant notice: mathematics teaching through the school olympiad problems is based on the principles of developmental education and is effective in terms of the use. A lack of the school olympiad problems, which should be included in the teaching process, creates some barriers for pupils during the training process to learning a full-fledged mathematics course in the secondary school.

The quantitative and qualitative studies of teaching material worldwide confirm that the secondary school mathematics courses are overloaded with the studied topics and appropriate exercises in almost all countries, due to which, teachers are not able to teach solving the mathematical olympiad problems to the whole class. As a rule, these problems are solved at the sessions of the elective or mathematics subject sessions, i.e. only with a small part of the class. Therefore, the pupils, with relatively better performance in the traditional school course get more mental workload developing a thoughtful outlook, than the average and weaker pupils. The reason for this is that the vast majority of school graduates cannot even solve the simple algorithmic problems.

The general methods of solving the problems have been developed, but less attention was attached to the private-methodological aspects, as well as the methods of solving problems of some classes have been developed, which are not systematized, because classification of a system of the mathematical olympiad problems is not carried in the secondary school, and there are not developed their teaching

methodological features for their solving, which are associated with the specificity of the strengthened mathematics teaching system, which is primarily due to the fact that teaching of high school students specialized in mathematics does not include sufficient attention to the theoretical and methodical features of solving the mathematical olympiad problems.

In order to solve this problem, we consider it necessary to:

- define the essence, specificity, purpose and function of teaching the school mathematical olympiad problem solving;
- develop classification criteria for solving the school mathematical olympiad problems;
- develop special methodology for teaching the school mathematical olympiad problem solving.

The study of the problem of teaching the school mathematical olympiad problem solving comprises the numerous aspects, in particular: psychological, general and special methodological aspects, etc.

When studying the psychological aspects, it is especially important to know the specificities of the process of thinking at the different age stages of mental development of pupils, and what are the possibilities of acquiring knowledge with an adolescent of a given age. Georgian psychologists D. Uznadze, R. Natadze, Sh. Nadirashvili, I. Kotetishvili, Z. Vakhania and others have focused their research works on this problem. From the studies carried out in the Soviet and post-Soviet space, we can highlight research works performed by S.Rubinshtein, E.Kabanova-Meler, I.Yakimenskaya, V.Kruteksky, I.Kalyutkina, N.Menchinskaya, N.Talyzina, L.Fridman, A.Rakhimov, D.Yelkonin, V Davydov and others. Among the Western researcher, a great contribution to this field has been made by J. Piaget, He is an author of the original theory of child cognitive development. A great contribution to the studies of this issue was made by the American psychologist J. Bruner, who has proved that the child is always able to learn more than it seems at first glance, if we transfer him/her teaching material in accordance with the form of his/her thinking.

The special-methodological aspects of teaching the solving the school mathematical olympiad problems are less developed. Research works focused on this issue belong to D.Poya, AStolyar, T.Moralishvili, N.Alfutova, G.Berdzulishvili, G.Bregadze, B.Bakuradze, V.Litvinov, I.Perelman, V.Prasolov, A.Sgibiyev, A.Spivak, N.Oniani-Saginadze, G.Jinjikhadze, M.Lomtadze and others.

In the recent past, not a single dissertation addressing this topic was defended in Georgia and in the post-Soviet area. The fundamental studies of the issue of teaching solving mathematical olympiad problems have been carried out by G. Berdzulishvili and G. Bregadze. They have also developed the systems of problems forming the practical skills of solving the mathematical olympiad problems in the secondary school. In addition, only some publications address teaching any particular technique for problem solving, which creates an impression that the development of general and special-methodological basics methods for solving the school mathematical olympiad

problems poses some challenges. Such considerations are wrong and far from the truth.

These considerations clearly indicate relevance of a chosen topic of Doctoral thesis "Methodical Specifics of Solving Some Types of the School Mathematical Olympiad Problems"

In order to attain the objectives we have set, there have been identified the need for including the school mathematical olympiad problems in teaching process, the purpose of teaching them and function, and special methods and techniques for solving some types of the school mathematical olympiad problems have been developed, as well as the classification criteria of these types of problems and special methodology for teaching them, which is described in the second chapter of this dissertation and in the scientific publications of the author of dissertation. The teacher can use them in the ordinary teaching process by involving the olympiad problems, and they do not require additional training, are effective with their developmental functions, increase the level of pupils' mathematical training and raise i.q. scores in general, as had been shown by the conducted educational experiment.

Research subject. The research subject the development of methodology for teaching the school mathematical olympiad problem solving. To this end, we have studied and critically analyzed scientific-pedagogical and methodical literature related to the problem. As a result of a survey of pupils and discussions with practicing innovator-teachers, we have established that there have been developed theoretical and methodological bases of teaching the school mathematical olympiad problem solving. The existing methodological auxiliary schoolbooks published for teachers and exercise manuals published for pupils do not adequately address theoretical and practical issues of teaching the school mathematical olympiad problem solving.

Scope of research. The scope of research is the development of special methodology for teaching the school mathematical olympiad problem solving and working out the system of the school mathematical olympiad problems, which we consider appropriate to include in the mathematics classes at the second and third levels of secondary schools, when teaching particular topics.

We have selected issues that are in full compliance with issues to be studied in secondary school. From the issues under consideration, we have chosen and developed the following topics:

- methodical specifics of teaching some special techniques for solving the trigonometric equations and inequalities of olympiad content;
- solving some types of the mathematical olympiad problems using the interdisciplinary relationships;
- methodical specifics of teaching some non-standard techniques for solving the mathematical olympiad equations in a set of integers;
- solving some types of the algebraic olympiad problems through the geometric interpretation and their teaching methodology;

• methodology for teaching some special techniques for solving the function-related mathematical olympiad problems.

Goal of research. Through the observations on the teaching process in secondary school, generalizing the colleagues and own educational experiences, and by conducting an educational experiment, the research is aimed at increasing the level of education and training work by teaching the school mathematical olympiad problem solving at the second and third levels of secondary school, as well as at working out the systems of the school mathematical olympiad problems and developing their teaching methodology.

The goal of teaching the school mathematical olympiad problem solving is to develop such pupils' skills, such as searching and processing independently of new information beyond the received one.

There are some patterns in teaching the school mathematical olympiad problem solving, which have been identified by methodologists, mathematicians, teachers and psychologists, which are not systematized. From them, we have highlighted some points to be used for the topic of dissertation and have developed their special methodology for their teaching.

For pupils, a conscious attitude towards education and training, improvement of the moral and intellectual formation and civic education represent the main goal of the secondary school, towards which it is necessary develop pupils' skills of logical thinking, cognitive activity, ability to perceive reality objectively, which requires special mental training for pupils, that is why in full respect for the principle of didactics required for teaching material, we have chosen such systems of the school mathematical olympiad problems, the teaching content of which is fully adapted to the age of the pupils, and through the school mathematical olympiad problems, the deepening and expansion of teaching materials is carried out in accordance with pupils' intellectual level.

Novelty of research is that

• special methodology of teaching the school mathematical olympiad problem solving has been developed;

• it has been justified that teaching the school mathematical olympiad problem solving increases significantly the effectiveness of teaching process at the second and third levels of secondary school;

• inclusion of the systems of the school mathematical olympiad problems in teaching process facilitates attention of pupils not only to their mathematical studies, but also increases pupils' interest is studying the natural science-mathematical disciplines;

• for the mathematics courses for the second and third levels of secondary school, there have been worked out the systems of the school mathematical olympiad problems, the inclusion of which in teaching process is desirable when teaching particular topics and/or solving techniques;

• based on the analysis of psychological and educational patterns, relevant methodological recommendations have been developed.

One of the most promising ways to improve mathematics teaching in secondary school is the creation of a system of in-depth mathematics teaching. Creating the in-depth mathematics teaching system is associated with extracurricular activity in mathematics, whose theoretical and methodical grounds have not been developed yet. Issues related to mathematical extracurricular activity are perfectly worked out in this dissertation. We consider extracurricular activity to be a form of teaching, we also reviewed the goals and objectives of extracurricular activity, and compare it with teaching process in class, developed the methodological bases and forms of extracurricular activity, and we studied the purposes of the mathematics elective courses and mathematical circles and methodical specifics of their conduct.

Forms and methods of extracurricular activity that we studied, greatly enhance the level of knowledge of pupils, facilitate their general intellectual development, in particular the formation constructive and critical thinking, logic, intuition and improve the ability to express their own ideas, and so on.

Theoretical and practical bearing of research is that the use of research results in the practice of teaching increases the effectiveness of teaching process, acquiring proven knowledge by pupils and their interest in mathematical studies. Theoretical importance of research is expressed in a scientific concept of purposeful conduct of teaching process, and its practical importance involves introduction of a number of methodological innovations in mathematics teaching, in particular, it has been confirmed that inclusion of the school mathematical olympiad problems in teaching process strengthens pupils' motivation to study mathematics, promotes the development of logical thinking, and raises i.q. scores.

The theoretical and methodological basis of the research is the philosophical and psychological provisions on unlocking and developing the intellectual capacities of the individual.

An educational experiment was conducted in LLC Kutaisi School No 1, LEPL Kutaisi Andria Razmadze Physics and Mathematics Public School No 41, LEPL Ioseb Otskheli Kutaisi Public School No 2, LEPL Kutaisi Public School No 34, LLC Kutaisi School "XXI Century", LEPL Tskaltubo village public school of Tskhaltubo municipality, Banoja village public school of Tskhaltubo municipality, LEPL Zestafoni Public School No 7, LEPL Sviri station public school of Zestafoni municipality, LEPL Rupoti village public school of Terjola municipality, LEPL Chkari Public School №2 of Terjola municipality, LEPL Samtredia Public School No 12, LEPL Rokiti village public school of Bagdati municipality, LEPL Salominao village public school of Vani municipality, LEPL Tkibuli Public School No 1, and in Kutaisi Palace of the Students.

The experimental study was carried out in two stages on three forms during four academic years. Prior to the educational experiment, we carried out analysis of works related to the issues discussed in this dissertation. During the experiment (2011-2013), we have produced list of the topics to be considered, and a special methodology of its teaching has been developed, which had consequently undergone insignificant changes in the work process. The educational experiment (2013-2017) has proven the

effectiveness of the inclusion of the school mathematical olympiad problems in educational process and the methodology for their solving.

The statistical evaluation of the educational experiment was made by χ^2 criterion. The pedagogical experiment demonstrated that inclusion of the methodologically rightfully chosen school mathematical olympiad problems in teaching process contributes significantly to increasing the effectiveness of teaching. Implementation of teaching process through this form and by methodology we developed, raises pupils' i.q. scores, develops logical thinking, increases their motivation and cognitive interests towards their mathematical studies.

The following results have been obtained during the process of theoretical and experimental research:

- Methodical specifics of teaching the school mathematical olympiad problem solving have been analyzed, and their practical importance has been scientifically justified;
- There have been developed the need for including the school mathematical olympiad problems in educational process and methodical specifics of teaching their solving when teaching particular topics;
- According to each sign, there have been singled out the appropriate system of the exercises, which can be reviewed with the class in the elective, mathematical circles and for individual working. For each class of problems classified in accordance with this sign, special methodology for their teaching has been develop, which involves review of the individual stages of solving, their sequence, using the appropriate types of mathematical models.

The following provisions and conclusions have been brought to defense:

- Inclusion of the school mathematical olympiad problems in educational process, as a tool for mental and intellectual development of pupils;
- Defining the role and place of the school mathematical olympiad problems in general education;
- Studying methodical specifics of solving the school mathematical olympiad problems;
- Working out the systems of the school mathematical olympiad problems;
- Experimental study of the developed methodology for solving the school mathematical olympiad problems.

Approbation of work and publication. Presentations on the issues considered in this dissertation were made at the university, republican and international scientific and pedagogical conferences. The results of the study were systematically introduced to the scientific seminar existing at the Methodology Department of the Faculty of Pedagogics of Akaki Tsereteli State University. The main results of the dissertation were also presented to the scientific seminar of the Faculty of Education of Iakob Gogebashvili Telavi State University. The paper was discussed and reviewed at a joint

session of the departments of Teaching Methodology and Pedagogics of Akaki Tsereteli State University's Faculty of Pedagogics.

Volume and Structure of Dissertation. Dissertation includes Introduction, two chapters, general conclusions, list of References and Annex. Introduction dwells on justification of goal and objectives of choosing the topic of dissertation, describes topicality of research, its theoretical and practical bearing, identifies the problem, general objectives and methods of research. There are also formulated the basic provisions. The structure of paper has the following form:

Brief description of work

The first paragraph of Chapter I refers to the history of formation and development of school mathematics. According to the findings, on November 3, 1933, the first mathematical olympiad for pupils was held in Tbilisi, which triggered such olympiads throughout the former Soviet Union. On November 28, 1933, in Tbilisi, the district olympiad for pupils was was held in mathematics. This venture has involved the big interest and attention of the Georgian public and the press. Local and republican newspapers devoted articles to this Olympiad.

The city school Mathematical Olympiad was first conducted in Tbilisi on July 18, 1934. The winners of the Olympiad were invited to the Tbilisi State University to the Olympiad, organized by Georgian mathematicians A.Kharadze, L. Gokieli, D.Dolidze, M. Koniashvili, and K. Sulakvelidze. The first Republican Mathematical Olympiad for young mathematicians in the Soviet Union was held in Georgia, in 1957, and the chairman of the organizing committee was Vladimer Chelidze, corresponding member of Georgian Academy of Sciences.

The history of the mathematical Olympiads for pupils continued so that in 1967 Georgia was again at the epicenter of the mathematical Olympiads for pupils. In this year, the first all-Union Olympiad in Mathematics was held in Tbilisi. In 1981, Georgia hosted again the all-Union School Olympiad in Mathematics. The Chairman of the Organizing Committee of this Olympiad was T. Gegelia, a corresponding member of Georgian Academy of Sciences. An active role in the successful organization of these Olympiads was played by famous Georgian mathematicians, academician I. Kiguradze, professors L. Gogoladze, Z. Chanturia, T. Chanturia, who were also then actively participated in the conduct of the Olympiads for pupils in Georgia.

The next mathematical Olympiad throughout the Soviet Union was held in Russia, where the first mathematical Olympiad was organized in Leningrad (now St. Petersburg) in the autumn of 1934 by Professor B.Dellone, a corresponding member of the Soviet Academy of Sciences. In the conduct of this Olympiad, there were also actively involved O. Zhitomirsky, V. Tartakovsky, D. Fadeyev and G. Fikhtengolts. This Olympiad was conducted in several stages (the open stage in absentia, then the writing attendance stage and conclusive attendance stage). The Olympiad was organized only for senior pupils. Later, since 1939, ninth graders have taken part in the Olympiad, and eights graders have also been involved since 1940. Since 1969, the olympiads are held for fifth-tenth graders. The peculiarity of the Leningrad Olympiad, which has been preserved so far, is that the final stage is conducted in oral form, where pupils communicate their solutions of problems to jury members. The first city Olympiad, following the Leningrad Mathematical Olympiad, was held in Moscow in 1935,. The chairman of the organizing committee was academician P.Aleksandrov, the organizing committee was composed of the well-known mathematicians, faculty members of Moscow State University, professors A. Kolmogorov, L. Lyusternik, L. Shnirekman, V. Kogan, S. Yanovskaya. The success of the first mathematical Olympiad encouraged the organizers of the Moscow State University to create a

school mathematical circle, after creation of which special literature for pupils and mathematicians is regularly created.

Shortly after the end of World War II, the Olympiads were restored. In the first years, the Olympiads were held only in large cities, where with top universities, and in the late 1950s and in the early 1960s, the mathematical Olympiads became traditional almost for the entire country. The Olympiads were conducted by universities and pedagogical institutions together with public education authority.

The idea of the school mathematical Olympiad has brought together the university faculty members, researchers from the research institutions, teachers of specialized schools, postgraduate students, and undergraduates who tried to identify talented young people and help them to improve their professional skills. This public engagement was supported by government in many ways, including financial.

In 1960, The first mathematical Olympiad involving several regions of the Russian Federation was held in Moscow. This Olympiad is often called the "zero" mathematical Olympiad of the Russian Federation, since the official numbering started in 1961. The first all-Russian Mathematical Olympiad involved almost all Russian regions, and there were also invited combined teams from the constituent republics of the then Soviet Union with 4 members in each, which were composed of the winners of the Mathematical Olympiads of the the constituent republics. In the early years, the all-Russian mathematical Olympiads were held in Moscow, and later the honor was conferred on other cities as well. The Olympiad became independent to some extent. Since 1967, the Olympiad has given the official name "The All-Union Mathematical Olympiad for Pupils".

During roughly the same period, there was held the first international mathematical Olympiad for pupils, which was initiated by the Romanian Mathematical and Physical Society. In the summer of 1959, seven pupils from Eastern Europe were brought together at the first International Mathematical Olympiad in Bucharest. Later, the number of countries participating in the Olympiad was increased. For example, 98 countries of the world participated in the International Mathematical Olympiad. During the existence of the Soviet Union, the Olympiads in the constituent republics, as well as all-Russian and Mathematical Olympiads allowed the organizing committees for selecting the representative combined team of the Soviet Union in the International Mathematical Olympiads, including combined team of pupils from Georgia, who have repeatedly represented the country with distinction. Traditionally, our pupils demonstrated very good results in the International Mathematical Olympiads for pupils, and they were among the leaders in all Olympiads in the team event.

A sharp increase in the number of participants of the mathematical Olympiads for pupils created some organizational difficulties in the conduct of Olympiads, so since 1975, some changes have been made in the structures of the all-Union Olympiads, which is mainly reflected in the competition stages of Olympiads, and the number of participants in the final stage has been reduced. On the way to the peak, the young mathematicians were required to move beyond the school, district and/or city,

regional, republican and the all-Union stages. Pupils winning the prize-places in each round of the olympiads were qualified for the next round. Thus, a fairly rigorous system of picking the Olympiad winners has been established.

After the collapse of the Soviet Union, since 1991, the mathematical Olympiads had been held in the former Union republics, including mathematical Olympiads for Georgian pupils. At present, Mathematical Olympiads for Georgian pupils are conducted in the following phases: school, district and municipal, regional and republican phases.

All combined teams of Georgia (Mathematics, Informatics, Physics, Chemistry, Biology) were awarded with three gold, 12 silver, 57 bronze medals and 48 honorary certificates during the last 10 years in International Training Olympiads. Members of Georgian team are selected for the Mathematical International Training Olympiads on the basis of the results of third qualifying round of the National Training Olympiad, as well as on the results of all qualifying rounds..

Goals, objectives, rules and structure of conducting the school mathematical olympiads are considered, and it has been noticed that one of the most important goals of conducting olympiads is to motivate pupils to study mathematics, and to involve them actively in in-depth mathematical studies that is carried out through training of pupils in the elective course in mathematics, extracurrcular mathematics activity and by involving them in the mathematical subject circles. Pupils like to participate voluntarily in the competitions, the environment in which the olympiads are conducted, as well as the principle that „the most important thing is to participate and not to win". Pupils' sporting blood is also important in the olympiad. The Mathematical olympiads fill pupils with a high competitive spirit and encourage them to compete with their peers to in solving the olympiad mathematical problems.

Conducting mathematical olympiads for pupils serves identification and further enhancement of their mathematical capacities. Mathematical olympiads are assisting the professional development of a school mathematics teacher. In order to prepare and conduct the Olympiad, mathematics teacher has to create a mathematical subject circle of pupils, undertake highly labor-intensive preparatory work, select and solve different mathematical problems, get acquainted with various mathematical issues, new mathematical and methodical literature. For the work of the mathematical subject circles and requires and mathematical Olympiads for pupils, it is required to collect materials and then to process them, and teaching process for math teacher is a form of scientific-methodical activity, because the work of the mathematical subject circles and selection of problems for mathematical olympiads for pupils require a quite thorough knowledge of mathematics and its teaching methodology. Head of a circle is required to be able to carefully consider the methodology of solving each problem during the circle classes. During the circle classes, it is necessary to expand the studied mathematical material and provide the -depth study of the issues under consideration, which sometimes goes beyond the school curriculum. In order for mathematics teacher to discuss such issues, he/she is required to have in-depth knowledge of materials to be transferred, as well as to be highly knowledgeable of methodology of teaching

materials to be transferred to pupils. On the other hand, the leadership of mathematics subjects,,

Mathematical school olympiads are summarizing the state of teaching mathematics in the school, district, region and in the country as a whole. The international mathematical Olympiads allow for looking out for dynamics of the level of mathematical education in different countries. The identification of changes is of particular importance, because in modern conditions, it allows countries participating in the olympiads for taking immediate measures and making changes in the content of mathematical education of pupils, to which training of future mathematicians is closely linked.

The second paragraph dwells on the interrelation between the mathematical olympiad problems and problems to be examined in mathematics school curriculum. Our position is clear, we tend to the intuitive understanding of the concept of problem. Each problem consists of two parts:

The terms, which constitute the combination of objects and the relationship between them, is given in the description and request (question) of what is to be found by the terms. As to the question of how the task is asked, you can determine what the answer can be, what it should be. For this purpose, we have divided the three classes.

We understand the solution of the problem to mean a kind of meaningful action-process that results in the outcome that satisfies the statement of problem. The process of solving the problem consists of subprocesses such as:

- Discovery of problem-based situation;
- Setting of the problem: elements of the statement of problem and identification of the relationships between them and the need (goal);
- Finding the way for problem solving;

Two strategies of problem solving have been examined: direct search and inverted search.

3. With a direct search, the pupil tries to solve the problem by any method, and then observes whether the method used has moved the process of solving the problem forward.
4. With an inverted search, the pupil observes the obtained solution and raises questions: What is the initial step that should be taken to arive at the solution to problem? After finding such a step, it is necessary to determine the step immediately after the first step, then the following step, and so on, until we arrive at the solution to problem.

The process of an inverted search is based on the comparison of goal and resources: at each step, the intermediate goal is comparing with the final one, and the operator is finding, which reduces the length of the way of problem solving. There are two types of operators - algorithmic and heuristic: the algorithmic operator (a set of rules leading to a guaranteed result); the heuristic operator (intended for the rather complex problems, for which the algorithms have not been found or no exist).

Mathematical problems are divided into standard and nonstandard problems. Problems, whose solving algorithm is known are considered to be the standard ones,

but the algorithm for solving the non-standard problem is not known. The olympiad problems include the class of nonstandard problems. We understand the non-standard problems to mean a much broader class, particularly those problems that are not standard. The approaches of various authors to the phases of problem solving have been examined and relevant methodological conclusions have been made.

We believe that the school mathematical olympiad problems should be included by teacher in the class process when considering particular topic. We want to emphasize that we do not ask for the special time to do this. When discussing the study topics, pupils should be familiarized with the topic-related olympiad problems, and besides, the process of teaching should be carried out so that the age peculiarities of pupils are taken into consideration.

Inclusion of the school mathematical olympiad problems in the class educational process is characterized by some peculiarities, particularly,

1. Teaching the school mathematical olympiad problem solving should be carried out when solving the particular problems;

2. In most cases, the teacher should not teach special methods and/or techniques for solving the school mathematical olympiad problems as some kind of blueprint, and he/she should use such instructions and questions, which will allow pupils themselves for coming to the "discovery" of special method and/or technique;

3. The teacher should carry out the process of teaching the school mathematical olympiad problem solving so that the whole class is involved in it, and the pupils should have the opportunity to express their views, even if probably these views are wrong, and to follow-up their own approaches. Teachers should respect the views expressed by pupils;

4. After solving the school mathematical olympiad problem, teacher should carry out comprehensive methodological analysis of used method and/or technique, and if the type of a solved problem allows for such possibility, to generalize it in relation with problems of a much broader class;

5. In the subsequent phases, when pupils gain sufficient experience, they will succeed in using the methods and/or techniques for solving the school mathematical olympiad problems, the intervention of the teacher should be reduced both when choosing method and/or technique, and during the process of solving. He/she should shift his/her attention to those pupils, for whom it is relatively difficult to use methods and/or techniques of solving.

The third paragraph – "Extracurricular activity, methodological and methodical foundations of its conduct" - contains six sub-paragraphs. In the first sub-section, we consider extracurricular activity in mathematics as a form of teaching. We examine extracurricular work as an integral part of the school's unified educational-training work, which is one of the active forms of organizing leisure time for pupils, provides a wide range of opportunities for the multifaceted development of pupils to develop and preparing them for life. Extracurricular activity involves various activities and promotes the process person's education and upbringing.

Extracurricular activity in mathematics is not compulsory for all pupils, but it is desirable that the teacher systematically conducts extracurricular lessons, which will have different loading. In particular:

a) supplementary classes for slow learners;
b) working with pupils, who demonstrate mathematical skills, have the gift and interest in their intense mathematical studies.

There should be a close interconnection between activities carried out by the teacher during the class and extracurricular activities. Class sessions arousing pupils' interest in the subject studied, necessitate the conduct of the extracurricular lessons, and vice versa, on the extracurricular lessons, pupils enhance and deepen their knowledge of the subject studied, learn about skills of practical application of knowledge that improves academic performance of pupils and increase their interest in their mathematical studies. It should be especially emphasized that it is unacceptable that extracurricular activity is duplication of classes.

There have examined in detail goals and objectives of extracurricular activity in mathematics, forms of the conduct of classes and their methodology. Factors of the pupil's attitude towards the subject studied have also been considered: with personality characteristics (teacher, pupil); with particularities of the subject matter; with methodology of the subject matter and its practical teaching.

ოზეშბThere have been identified different categories of pupils varied one from another in terms of their mathematical studies, and the conclusion has been made that when developing math teaching methodology, the interests of each group of pupils should be taken into consideration. It is necessary to develop the forms and methods of work of mathematics teachers with each group pupils in both classes and extracurricular activities. There have been identified issues to be solved during the extracurricular lessons and ways to address them have been defined. The conclusion is that extracurricular activity gives wider opportunities for creative activities.

The second sub-paragraph dwells on a comparative analysis of the mathematics extracurricular and class teaching processes. There have been highlighted the general and distinctive features of these both processes, and the relevant methodological conclusions have been made.

The third subparagraph describes the form of mathematical extracurricular activity in detail, particularly: mathematical subject circles for different age groups (elementary, basic and secondary school); mathematical competitions, quizzes, contests, the competitions between the clubs of cheerful and sharp-witted persons; mathematical thematic hours (conversations, lectures); mathematical mornings and evenings; mathematical performances; mathematical Olympiads; faculty elective mathematical courses; mathematical excursions; mathematical weeks (decades); printing (handwritten) mathematical newspapers (mathematical subject circle's newspapers); extracurricular reading of popular, scientific and literature of mathematical content; mathematical abstracts and essays; school scientific mathematical conferences; mathematical model designing, etc.

.We believe that mathematical extracurricular activity and the examples discussed in class should meet certain requirements, particularly: be diverse; the should also be chosen by considering the age peculiarities of pupils; they should be designed for different categories of pupils: for those who have special mathematical skills, and, at the same time, are interested in mathematics, and those pupils, who have not yet expressed any special interest in mathematics; Extracurricular activity should be very different from the form of the conduct of lesson and from other compulsory measures: extracurricular activity should also be based on the voluntary principles, and the lessons should be held either after classes, or in the evening hours.

These requirements are sometimes not fulfilled or are not paid due attention. Studies have shown that violation of the basic rules leads to the disastrous results that the mathematical subject circles in the schools in schools are often disbanded. Mathematical competitions and mathematical mornings-evenings are very small, it is seen in a glance they are conducted because they must be conducted, and pupils are less active. Therefore, we believe that when organizing them, it is important not only to think seriously about their content, but also, we consider it indispensable to know the forms, especially methodology of their conduct. On the extracurricular lessons, it is necessary to use those problems, whose solving methods would be interesting and satisfying the interests and requirements of all pupils attending the lesson.

Extracurricular activity can be successfully used to enhance the knowledge acquired on the lessons envisaged by a school mathematics curriculum. In school, it is necessary to pay particular attention to extracurricular activities in mathematics, because mathematics extracurricular class for pupils is often the first phase of extensive mathematics teaching, which subsequently contributes to pupils' involvement in the faculty elective mathematics course, or pupils' enrolment in mathematics class, and then their enrolment in the mathematics specialty at the University.

The fourth subparagraph describes the mathematics classes and the methods of their conduct. The main goal of the mathematics faculty course is to deepen and enhance acquired knowledge in the subject studied, arouse interest of pupils in their mathematical studies, develop their mathematical skills, to create a palate for the topics of independent pupil's math classes, to develop their initiatives and creativity, mental education, and so on. The basic course of mathematics, along with elective faculty course, is a mathematical enhanced course for the given class.

The mathematics elective faculty course should be drawn so that all the questions that will be studied in the course of the school mathematics are synchronized during the study of the course. In the case, when the basic mathematics class is carried out by one teacher, and the faculty course is carried out by another teacher, the subject of discussions on the faculty course classes is independent of the main course topics from the mathematics curriculum (in this case, classes on the topics studied somewhat later than the basic mathematics class). It should be noted that the study of the topics discussed in the elective course must not precede the study of similar topics in the main school. Signing up of pupils for the faculty elective courses is carried out on

voluntary basis, in accordance with their interests. The faculty elective courses in mathematics should be designed in a way that makes them interesting and attractive to pupils, and they must have a developmental function as well. The main forms of their conduct are lectures, seminars, discussions, problem solving, pupil's abstracts, which may relate both to theoretical issues of mathematics and solving the problem cycles, mathematical essays, pupils' scientific reports, and so on. Notwithstanding the form and method of the conduct the faculty course class, its purpose is unchanged, it serves the transfer of the key issues of mathematics chosen elective course and its teaching.

It should be taken into consideration that teacher should not give preference to any form or method of explaining the issue. It should be constantly remembered that during the mathematics faculty elective class, we should give priority to independent activities of pupils, for which we should often include in the topic of the faculty elective class - problem solving, abstracts and presentations, seminars and discussions, reading of mathematics teaching and scientific-popular literature, etc. During the mathematics faculty elective class, it is advisable to use the method of problem-based learning. In the mathematics faculty elective class course, we should offer pupils theorems in the form of problems. If theorem to be proved by pupil is large in volume or complex, then it should be broken down into several problems, so that the solution to previous problem serves to solve the following problem, definitions, which help pupils in problem solving, or are involved in the text of problem or specifically stated by teacher. If necessary, before starting to solve problem, teacher conducts the necessary conversation in advance. The teacher distributes to pupils the previously printed assignments individually on each lesson. It is also beneficial to use the problem-framing process.

In order to conduct extracurricular activity, we should select non-standard problems in accordance with the strength of the group by involving the olympiad problems, for which it is desirable to use problems in accordance with the different topics and on the basis of school Olympiads of different levels. It is also possible to involve problems designed for the work of for the mathematical subject circles. A good outcome is produced by search and use of problems from the Internet sites.

The main thing is to convince slow learners on the extracurricular lessons of mathematics that they can solve the offered problems. For this, at least in the beginning, the teacher is required to help the learner, and then this help should gradually be weakened and learners should be able to independently solve the offered problems.

The fifth sub-paragraph dwells on the lesson of the mathematical subject circle methodical specifics of its conduct. For conducting the lesson of the mathematical subject circle, we use the well-tested in practice method, which is associated with delivering problems to the pupils in the form of the printed assignments. At the same time, there are printed only problems intended for one class. The number of printed problems on the sheet should be 8-10, and they must be selected so that pupils can find their solution during one class. The maximum duration of the mathematical subject circle's lesson should be two academic hours.

During the first two classes of the mathematical subject circle, the head is required to assess the overall level of pupils' mathematical training, as well as their preparednes according to individual topics. To this end, it is recommended that the teacher at the beginning of the mathematical subject circle, during the so called "zero" lessons, the teacher is required to conduct written lessons, which will involve problems selected from different topics, and after correcting the mistakes, there should be created the statistical data reflecting not only all pupils' performance in accordance with topics, but also performance obtained as a result solving the problem by each pupil. After processing the statistical data, the teacher has also a clear idea of the level of mathematical training of members of the mathematical subject circle, which will then help the teacher to select problems and particular topics of study to be considered on the lessons.

At the beginning of the lesson of the mathematical subject circle, the teacher explains a new topic, from which then the problems will be taken and then they will be given to the pupils as a printed assignment. When transferring these topics, the teacher of the mathematical subject circle should follow some methodological approaches. In particular:

At the beginning of the lesson of the mathematical subject circle, pupils should not only be acquainted with theoretical material around the topic, but also the teacher should solve two or three the problem of average complexity. These problems should be selected by the teacher to demonstrate a new method or/and a new of way of problem solving.

Some sheets of assignments for the lesson of the mathematical subject circle may also contain the formulation of some theorems from any particular topic. The practical applications of these theorems are advisable to be shown at the beginning of lesson when solving problems. Approval of these theorems is not obligatory at the beginning of the lesson, moreover, sometimes this may even be counter-productive, because frequently, the process of proving the theorem takes most of the time allocated to class. Some of the theorems written on the sheet are proved within the mathematics school courses. Proving of some theorems is considered to be a specific problem. Sometimes, in order to solve such type of problem, it is necessary to solve several preceding problems, so we encounter such problems at the bottom of sheet. It is worthwhile to consider that first of all, it would be better not to understand the theorem's proving details, but the to use this theorem in the process of problem solving.

In the course of circle activities, the pupils are solving the problems given them as assignments, and trying to get a credit from the circle's teacher. There may be several teachers on the lessons of the mathematical subject circle, It is desirable to attain the goal of having no more than seven pupils in each class, because the teacher has to deal with each student individually to solve problems. If the pupil has solved problem correctly, then it is advisable to congratulate the student on this, and to put a "plus" mark in a special table. If the pupil has solved problem incorrectly, then it

would be preferable that the pupil continues solving this problem. Sometimes, it would be advisable teacher gives some instructions to the pupil.

The main purpose of the subjects of a mathematical subject circle is to let the pupil feel the joy of solving mathematical problem, and on that basis to develop of quick-wittedness of pupils and increase their stock of experience. Therefore, due to our practice, we strongly oppose the lesson of the mathematical subject circle. making assessments and marks; organizing review written works on the lessons of the mathematical subject circles; making compulsory pupils' attendance in circle activities; except in special cases, giving instructions to all problems that are printed on the sheets. The pupils are very happy and their interest is increasing when they solve problem independently. The problem solved due to someone's idea and instructions can not make them happy. We do not consider it necessary to give homework assignment to members of the mathematical subject circles. We do not consider it to be reasonably appropriate to set goal to the mathematical subject circle to prepare for any school Olympiad, or for any kind of mathematical competition, despite the fact that practice confirms that members of the mathematical subject circle achieve much better results than on the subject Olympiads than pupils of regular schools.

To select the problems, the teachers of the mathematical subject circle should be guided primarily by their own taste and by the age and mental abilities of their pupils> They have to always remember that the content of problemss and the ways and/or techniques of problem solving should be preferable to teachers first, and at the same time, the process of their solving must be interesting and accessible for pupils and their solutions.

The sixth sub-paragraph describes the organization and methodology composing the school mathematical Olympiad problems. Composing the Olympiad problems is quite difficult and it involves persons with a very high level of mathematical knowledge. Mathematics has its own aesthetics, ethics, harmony, music, beauty and all this is taken into account when composing the problem. As a rule, the process of composing the school mathematical Olympiad problems involves people, who have themselves participated actively and successfully participated in the mathematical Olympiads, but formally, this is the prerogative of a special Methodical Commission. After composing the problems, their solving is carried out by other members of the Commission, and there is defined the complexity of these problems. After that, these problems are discussed, on the basis of which, there will be identified the best one among them. The Commission members work out the recommendations on which class and which level of assignment they should be included in. This process envisages a certain harmony, and when composing these problems, each version should include problems of various complexities from the different topics of the school curriculum of the appropriate class.

Half of the members of the Commission are those bachelor, master and PhD students, who are the winners of various high level mathematical Olympiads during the school years. The second half is represented by the university faculty members and

employees. In recent years, the school mathematics teachers are very rarely involved in the methodical commissions. Composing the school mathematical Olympiad problem is not a small scientific work, it will not be reimbursed financially, but its reimbursement is only the author's love for mathematics and the smiling and happy faces of pupils participating in the Olympiad, some of whom may then become the colleagues even problems' composer.

At the end of this chapter, there are given some conclusions and methodical recommendations as follows:

1. In mathematics school teaching process, there is imbalance in terms of time distribution between teaching theoretical issues and problem solving. To cope with this problem, we consider it necessary that during transferring theoretical issues to pupils in math class, a particular focus should not be turned to strict proving of mathematical theorems and going into detail on the process of proving. An explanation of issues should be based on asserting the mathematical facts and provisions, after which the knowledge-building process should be conducted through the problems specifically selected for the particular topics. We believe it is reasonable that proving the school course mathematical theorem or provision should be transferred to pupils in the form of problems, which raises the level of pupils' knowledge and enhances the mathematics application capabilities in terms of both interdisciplinary and intrasubject relationships.

2. Increasing the effectiveness of teaching is caused by planning of teaching process the so that at least two-thirds of the mathematical teaching time should be devoted problem solving than studying theoretical material. Our recommendation is to include the theoretical issues in the process of teaching in the form of problems and to renounce going into detail on theorems and provisions by strict mathematical judgements. In addition, the mathematical school course teaching through the selected mathematical problems should be carried out only during problem solving, and it should be taken into account that the problem cannot be solved only by solving large number of them, and of particular importance are the problems' content and techniques of their solving. The set goal is effectively resolved in math class process, in accordance with the particular topics, by involving the school mathematical Olympiad problems.

3. The teacher is required to build the process of teaching the school mathematical Olympiad problem solving on the scientific content of teaching, and on conscious comprehension, which is based on active memorizing and logical thinking, because the full development of the human person from the pupil, along with a number of other activities, requires such teaching, which provides gradual comprehension of the foundations of science.

4. The task of the creative thinking of the pupils is strongly influenced by problems, are not directly solved by using the known algorithms. The process of solving such problems requires cognitive thinking, which enhances the joy of finding the way of solving the problem, which is an emotional factor and is a powerful tool for the pupil's behavior. Proper management of emotional factor is of utmost importance

in all areas of human activity, as well as in the process of forming a well-integrated personality from the pupil and in the process of teaching of him/her. Emotions and experiences of the pupils, as exponents of their subjective state, are mostly associated with the teaching process, so giving the right direction to emotions, and simultaneously the use of them as psychological factor to cope with problem, first and foremost is the prerogative of school and teacher. The qualitative transformation and development of emotions of pupils are mostly carried out through education and teaching. Emotions caused by teaching the school mathematical olympiad problem solving should be used as a part of the educational process. Therefore, the teacher should take into account the specific ways and measures that enable the pupils to develop positive emotions and feelings, and at the same time, will make teaching process attractive and fun.

5. Problems in the mathematics school textbooks correspond to the scheme "theory → problems". We believe that it would be better to use in teaching process the scheme „problems → theory → problems", which will establish the principles of developmental teaching in the secondary school teaching practice, and will allow us not only for acquiring theoretical material, but also for helping the pupils to acquire independently new knowledge.

6, Establishing the scheme „problems → theory → problems" in teaching mathematics in secondary school is carried out effectively, when along with the math class, the pupils are also actively involved in extracurricular activities: are brought together in the mathematics subject circle according to their age, involved in the mathematical competitions, quizzes, contests, in the competitions between the clubs of cheerful and sharp-witted persons, attend the mathematical thematic hours (conversations, lectures); mathematical morning and evenings, are involved in mathematical olympiads, attend the math elective courses, organize mathematical excursions, mathematical weeks (decade), publish printing (handwritten) mathematical newspapers (mathematical subject circle's newspapers); arrange extracurricular reading of popular, scientific and literature of mathematical content, write mathematical abstracts and essays, organize the school scientific mathematical conferences, design and prepare mathematical models, and so on.

The second paragraph of the second chapter deals with solutions of the olympiad-type trigonometric equations and inequalities. Such trigonometric equations and inequalities with their form and content can be involved in the teaching process of secondary schools, there is no need for additional teaching time and they are easily understood by pupils. Their involvement in the teaching process without any artificiality occurs naturally, the process of solving is very simple and allows pupils for seeing the originality of the techniques of solving trigonometric equations and inequalities and the beauty that accompanies solving such problems. Let's consider some of them. The solutions for trigonometric equations and inequalities are discussed using numerical inequalities, comparing the function domains, using the boundaries of functions, based on the properties of the functions sine and cosine, solving the

superimposition and some other types of trigonometric equations and inequalities. There have also been made relevant conclusions.

The second paragraph of the second chapter dwells on solving some types of the mathematical olympiad problems using the interdisciplinary relatiuonships. The process of establishing the interdisciplinary relatiuonships is considered to be a cognitive activity, which is carried out in three stages: 1. Preparation stage; 2. Resultant stage; 3. Strengthening stage.

The existing mathematics curriculum no longer envisages teaching materials with algebraic and geometric content separately, but to be fair, we must admit that the methodological approaches and content of teaching the algebraic and geometric materials are completely different.

Regulation of the educational process takes place directly on the lesson. The management of the implementation of the interdisciplinary relationships in the educational process is carried out through the targeted, informational and stimulating channels. The process is controlled by the teacher. The control and regulation of the educational process involves they organization of cognitive activities of pupils in the course class process.

In order to establish a methodological system of cognitive mathematical problems of an interdisciplinary nature, we have used the basic principles of constructing the didactic system of cognitive problems. Its peculiarities are as follows:

1. As the basis of problems typology, there have been taken those topics, which contain the interdisciplinary relationships.

2. The selection of the content of problems was carried out so that the meaning and result of problem solving expands the boundaries of the concepts, which are studied in related academic disciplines.

3. In each problem, there is prepared a system of questions for students to understand the interdisciplinary content of problems and to activate their knowledge acquired from other disciplines.

For the purpose of composing the existing types of problems, we have used the methodology developed on the basis of algebraic and geometric teaching materials and on the developed teaching methodology.

I. Within the mathematics course, we discussed the topics containing algebraic and geometric materials, which we assumed to be the topics containing the interdisciplinary relationships.

II. From the topics selected within the mathematics course, we highlighted the one that can be used to compose cognitive problems.

III. By using the interdisciplinary relationships, we have created the systems of problems, on the basis of algebraic and geometric materials.

IV. The aim of the system of developmental problems for activation of pupils' knowledge is to acquire the knowledge of algebraic and geometric material, and to strengthen this acquired knowledge, which is received by pupils in the process of solving cognitive problems, and which contributes to the development of operational thinking.

There has been considered solving of some problems containing the interdisciplinary relationships, which are appropriate for math lessons, and examining problems with more complex content is advisable during the mathematical subject circles or faculty studies, since the complexity of the examined problems exceeds the complexity of problems to be reviewed in school. Typology of reviewed problems can be used for the development of active, independent, creative thinking of pupils. Knowledge of typology of problems by the teacher is an important prerequisite for implementation of developmental teaching.

The third paragraph of the second chapter examines some of the methodical specifics of teaching non-standard techniques for solving the school mathematical olympiad problems in a set of integers.

Equation solving in a set of integers or natural numbers, which contains two or more unknowns, sometimes are very difficult to solve by standard methods, which makes it difficult for pupils. We have reviewed the method of solving some sort of equations that are based on the evaluation of the expressions in the equations. For greater clarity, we shall say that solving the equations by assessing the expressions in the equation means that the expression in the equation is assessed by shortage or surplus, or by shortage and surplus, that is, the determination of the range of values of this expression, but when it is impossible, it means the determination of boundaries, by shortage or surplus, or by shortage and surplus. There have been examined concrete problems.

The process of problem solving in a set of integers and natural numbers in a presented form arouses the pupil's interest and facilitates the introduction and application of special methods of problem solving.

The fourth paragraph of the second chapter considers solving some of the algebraic olympiad problems through the geometric interpretation, as well as methodical specifics of their teaching in secondary school.

For teaching mathematics in secondary school, of high importance is to establish a scientifically justified correct correlation between theoretical and practical issues. Obviously, this problem should be resolved when solving mathematical problems. In the full compliance with the principles of didactics, there should be selected such problems, which will be in full compliance with pupils' age peculiarities, will deepen and expand the study material, as well as correspond to the level of intellectual maturity of pupils and, at the same time, have a developmental function. Creating the systems of such kind of problems is not easy and requires a very high professional training, from both methodical and subjective points. It is not disputable for specialists that problems with geometric content are the best tools for solving the set problems. However, we should also note that solving these problems in the schools working with the regular education program deals with great difficulties, which, in most cases, is associated with a low level of job skill training received by pupils, and, sometimes, with the professionalism of teachers as well. The majority of pupils have a negative attitude towards solving geometric problems in schools, which is primarily due to a low level of abstract thinking of pupils, and they are not able to establish the

relationships between the elements of specific geometric problems, even when they perfectly understood this theoretically, and they even formulate them in the form of theorems or properties. The pupils have radically different attributes while studying algebraic and arithmetic material. When solving the problems with algebraic and arithmetic content, they try to use theoretical material known to them and the previously solved problems. The pupils ask and consult with each other. In short, they are much more interested in solving the problems with algebraic and arithmetic content, they are doing this a lot better than solving the geometric-content problems.

Considering this situation, the hypothesis that the pupil's interest in mathematics would be strengthened by such olympiad-content problems, which are algorithmic in their contents, and produce far better results for their solving, and it is advisable to use geometric methods and approaches rather than algebraic methods. This approach aims to alleviate tensions for pupils that accompany solving geometric problems, and to show them the need for studying the geometric material that sometimes it is possible to solve easily algebraic problems using the geometric approaches. With such approaches, we can easily shift from the consideration of the algebraic olympiad problems to problems with geometric content, which allows for obtaining the highest possible results in this regard. To test our assumptions, we have solved the algebraic problems that require special geometric approaches to be solved, they can be involved in the teaching process during the course of the study, and they do not require additional training time and have a large developmental function.

The paragraph describes problems relating to various academic topics and their solutions, and formulates methodological recommendations for teachers, which will help them in carrying out teaching process in full compliance with the principles of didactics and will allow them for composing similar problems for the different topics.

It has been proved that the geometric interpretation of the algebraic olympiad problems, and then solving these problems using special methods and/or techniques significantly increases the level of pupils' mathematical knowledge, promotes their general intellectual maturity, particularly the formation of constructive and critical thinking, logic, intuition and improving the expression of own ideas, as well as the development of skills of independent creative activity, and plays a major role in the formation of integrated personality.

The fifth paragraph of the fifth chapter dwells on methodical specifics of teaching some special techniques of solving olympiad problems relating to function recovery within the school course in mathematics.

The school course in mathematics is mostly focused on studying the functions that are recorded in an analytical form, and then by using these expressions, there are determined the properties of these functions. We have discussed the olympiad problems with the inverted content, when there are given some properties of the function, and by these feproperties, we should recover the function, determine its properties and find analytical record of this function.

A period when the function's derivative and integral were studied in school, the pupils understood, in varying degrees, the essence of function recovery by means of

some given proerties of this kind of problem–finction, bu now, when the differential accounting elements are no longer studied in school, the teacher has to work more to make pupils familiar with the essence of the inverted problem set, but the welcomed fact is that the pupils express the special interest towards solving this kind of problem.

The methodological approaches to solving the function recovery problems are less developed, and besides, the number of developed such kind of specific problems is not too large, thus from the methodological standpoint, it is interesting to consider different types of such problems, and to work out methodological recommendations for solving this kind of problems. In the same context, we should also consider some types of sequences, since the seuqnce is a particular type function. As the same category we have classified some types of the mathematical operations. There has been methodically analyzed solving this kind of problems, and relevant conclusions have been made.

The last paragraph of the dissertation refers to the statistical assessment of the conducted educational experiment. The statistical assessment was carried out by χ^2 method. The results obtained are reliable and confirm the effectiveness of methodology we have developed, which allows for making the following conclusions:

1. Methodology, whose general description is given in the dissertation, can be used in general education secondary school mathematics teaching practice, as a special method of teaching mathematics;

2. Teaching the school mathematical olympiad problem solving described in this dissertation contributes to deepening and expanding the level of pupils' mathematical knowledge;

3. From the methodological standpoint, we consider it necessary to split the school mathematical olympiad problems in secondary school into categories of similarity and the definitive distinctions After splitting, we should sort the problems depending on the complexity. In class, extracurricular activities and during the mathematical subject circle sessions, we should consider the more complex problems from each group, and the relatively simpler problems should be given to pupils as homework assignment or for the independent work. It is unacceptable for us in the teaching process to give pupils the complex olympiad problems, because those problems, whose solving goes beyond the mental capabilities of pupils are causing fear in them and doubts about their capabilities.

The developed methodology allows for making effective the process of teaching the school mathematical olympiad problem solving, which will finally strengthen the pupil's self-confidence confidence and will their interest in problem solving, which was confirmed by the conducted educational experiment.

At the end of the dissertation, there are given general conclusions and methodical recommendations as follows

Based on the theoretical research, with our own pedagogical experience over the years, consultations with the innmovator-educationalists and the results of the conducted educational experiment, we can draw up conclusions and recommendations formulated as follows:

1. The teacher is required to include the school mathematical olympiad problems in the class teaching process according to the particular topics under study, for which there is no need for additional time teaching. The process of solving the school mathematical olympiad problems should be built on the scientific content and cognitive perception of teaching, which is based on active memorizing and logical thinking, since the teaching process should be based on consistent understanding of the foundations of science.

2. Acquirement of methods and/or techniques of solving the school mathematical olympiad problems requires an individual approach to teacher, taking into consideration their peculiarities, individualization of homework assignments and individual work, and so on. The object of teacher's observation should the entire process of teaching problem solving examined in class and given as a homework assignment.

3. The use of special methods and/or techniques for solving the school mathematical olympiad problems depends on the experience of the teacher as well as on methods and special techniques of problem solving methodical specifics. Methods and special techniques and special techniques of problem solving require scientific study. In our opinion, more attention should be attached to teaching the method of solving the school mathematical olympiad problems in the higher school within the minor training programs of educational sciences of mathematical academic programs and at the trainings for teachers, since we believe that all mathematics teachers at all three educational levels of secondary school are required to be knowledgeable of special methods of solving the school mathematical olympiad problems.

4. In the process of teaching, pupils should be thoroughly familiarized with methods and techniques used for solving the school mathematical olympiad problems. If possible, it is desirable to use different methods and/or techniques for problem solving, and simultaneously, it is also necessary to justify the advantages of methods and/or techniques used.

5. The own pedagogical experience, consultations with colleagues and the educational experiments proved that the methodology for teaching the school mathematical olympiad problem solving should meet the certain requirements, as follows:

- The process of teaching special methods and/or techniques for solving the school mathematical olympiad problems should be carried out through the particular problems when solving the problem.
- The teacher is required to use such questions, which will help the pupils to find the most promising way to solve these problems.
- Methodology for teaching the school mathematical olympiad problem solving should involve analysis of problem, which is aimed at full perception of information contained in the statement of problem and, on this basis, at finding the way to address this problem.
- Before considering a new method and/or technique of solving the school mathematical olympiad problem, the pupils should be familiarized with the essence and specificity of method and/or technique to be used, and if possible, we should provide its generalization. With the purpose of strengthening the new knowledge acquired after solving the problem, the pupils should be given the relatively less complex problems as homework assignments or for individual working.
- The should be familiarized with the advantages of special methods and/or techniques using in solving the school mathematical olympiad problems, in comparison with other methods and/or techniques for solving the same problems.

6. Besides the well-known trigonometric equations and inequalities in general schools, there are also numerous forms of trigonometric equations and inequalities of mathematical olympiad content, whose solving requires the use of special methods and/or techniques. There have been pointed out some of these types of trigonometric equations and inequalities, which can be included in the educational process in secondary schools, and they do not require to allocate additional training time and are easily understood by pupils. Inclusion of such tringonometric equations and inequalities in the educational process without any artificiality, occurs naturally, the process of solving is very easy and allows pupils for seeing the originality of the techniques of solving trigonometric equations and inequalities and the beauty that accompanies solving such problems. The different approaches to solving the trigonometric equations and inequalities of the equations are discussed using numerical inequalities, comparing the function domains, using the boundaries of functions, based on the properties of the functions sine and cosine, solving the superimposition and some other types of trigonometric equations and inequalities.

We believe that the teacher should be perfectly knowledgeable of methods and/or techniques for solving the above mentioned problems, which, as required and in conformity with his/her own convictions, will be included by him/her in in the educational process that produces a good developmental effect on pupils.

7. Based on the consultations with colleagues, own experience, and the results of the conducted educational experiment, we can conclude that the advanced, creative school teacher effectively involves the school olympiad trigonometric equations and inequalities in the educational process, and the pupils are ready to consider in class the relatively more complex olympiad problems in comparison with problems envisaged in curriculum, which have been selected by teacher so that their solving requires sharpwittedness and knowledge of special techniques of solving, while such problems cannot be solved using an ordinary traditional approach, or they require quite a long time for solving or implementing the complex transformations.

8. Our research on methodical specifics of solving the school mathematical olympiad problems using the interdisciplinary relationships, based on the management of intellectual activities of pupils, was carried out in two directions: the first one is the direct way, during which the management is of immediate nature, and is expressed in the purposeful formation of intellectual action; the second one is an indirect way, which implies the indirect management of intellectual activity through methods and

content of teaching, while teaching encourages pupils to independently identify the functions and structures of intellectual activity in a general content of educational activities

In order to create a methodological system of cognitive mathematical problems, we have used the basic principles of constructing the didactic system of cognitive problems. Its peculiarities are as follows:

1. As the basis of problems typology, there have been taken those topics, which contain the interdisciplinary relationships.

2. The selection of the content of problems was carried out so that the meaning and result of problem solving expands the boundaries of the concepts, which are studied in related academic disciplines.

3. In each problem, there is prepared a system of questions for students to understand the interdisciplinary content of problems and to activate their knowledge acquired from other disciplines.

For the purpose of composing the existing types of problems, we have used the methodology developed on the basis of algebraic and geometric teaching materials and on the developed teaching methodology.

I. Within the mathematics course, we discussed the topics containing algebraic and geometric materials, which we assumed to be the topics containing the interdisciplinary relationships.

II. From the topics selected within the mathematics course, we highlighted the one that can be used to compose cognitive problems.

III. By using the interdisciplinary relationships, we have created the systems of problems, on the basis of algebraic and geometric materials.

There have been developed methodologically the problems with interdisciplinary relationships, from which the group of problems has been singled out, the use of which is reasonable in math class, and the group of more complex problems, the use of which is advisable during the mathematical subject circles or in extracurricular activities.

An educational experiment proved that Ny using typology of the above-mentioned problems, it possible to develop active, independent, creative thinking of pupils. We believe that knowledge of typology of problems by the teacher is an important prerequisite for implementation of developmental teaching.

9. The central element of the methodology teaching the school mathematical olympiad problem solving is the identification and use of the heuristic information contained in them, which facilitates the discovery of way to solve the equation. Unlike the standard equations, specifics of non-standard equations determine different types of solutions. Equation solving in a set of integers or natural numbers, which contains two or more unknowns, sometimes are very difficult to solve by standard methods, which makes it difficult for pupils. We have reviewed the method of solving some sort of equations that are based on the evaluation of the expressions in the equations. For greater clarity, we shall say that solving the equations by assessing the expressions in the equation means that the expression in the equation is assessed by shortage or surplus, or by shortage and surplus, that is, the determination of the range of values of

this expression, but when it is impossible, it means the determination of boundaries, by shortage or surplus, or by shortage and surplus.

10. Critical analysis of the state of mathematics teaching shows that most pupils are not well-versed in techniques for solving the school mathematical olympiad problems, the curriculum hardly envisages teaching the school mathematical olympiad problem solving. In the full observance of the principles of didactics, it is necessary to select such school mathematical olympiad problems, which will be in full compliance with the age peculiarities of the pupils and will correspond to the level of intellectual development of pupils6 and at the same time, will have a developmental function.

11. The educational experiment demonstrated that the pupils' interest in mathematics was intensified by such olympiad-nature problems, which, on account of their content, are algebraic, for their solving, the much better result is produced by the use of the geometric methods and approaches rather than algebraic methods. With such approaches, we can easily shift from the consideration of the algebraic olympiad problems to problems with geometric content, which allows for obtaining the highest possible results in this regard. To test our assumptions, we have solved the algebraic problems that require special geometric approaches to be solved, they can be involved in the teaching process during the course of the study, and they do not require additional training time and have a large developmental function. There have been worked out methodological recommendations for teachers, which will help them in conducting teaching process in full compliance with the principles of didactics and will enable them to compose similar problems for the different topics.

12. The geometric interpretation of the algebraic olympiad problems and teaching the special methods and/or techniques of solving allow for making mathematics teaching a combination of the didactics-based knowledge and cognitive technique required for its acquisition.

13. By means of some properties of a function, the function recovery problems pertain to the category of the olympiad problems, there is a general algorithm for their solving. In such problems, there is required to find the analytical expression of function. The educational experiment established that solving the function recovery problems significantly increases the level of pupils' mathematical knowledge, promotes their general intellectual maturity, particularly the formation of constructive and critical thinking, logic, intuition and improving the expression of own ideas, as well as the development of skills of independent creative activity, and plays a major role in the formation of integrated personality.

14. The educational experiment proved the effectiveness of the methodology we developed for solving the school mathematical olympiad problems in the secondary school mathematics course, it can be involved in the educational process and is a special methodology of mathematics teaching.

The basic results obtained in the dissertation are reflected in the following publications:

1. Gokadze I., Bakuradze B., Berdzulishvili G. On one non-standard method for solving equations in a set of integers. Periodical scientific journal ,,Intelekti", №3(53), Tbilisi, 2015, pp. 25-28.
2. Gokadze I., Berdzulishvili G. On some techniques for solving trigonometric equations and inequalities. (Part I). Periodical scientific journal ,,Intelekti", № 3(56), Tbilisi, 2017, pp. 12-16.
3. Gokadze I., Berdzulishvili G. On some techniques for solving trigonometric equations and inequalities. (Part II). Periodical scientific journal ,,Intelekti", № 3(56), Tbilisi, 2017, pp. 17-21.
4. Gokadze I., Berdzulishvili G. The use of interdisciplinary relationships in solving the olympiad problems. (Part I). Periodical scientific journal ,,Intelekti", №1(57), Tbilisi, 2017, pp. 42-45.
5. Gokadze I., Berdzulishvili G. The use of interdisciplinary relationships in solving the olympiad problems. (Part II). Periodical scientific journal ,,Intelekti", №1(57), Tbilisi, 2017, pp. 46-50.
6. Gokadze I., Berdzulishvili G., Bregadze G. Geometric interpretation of the algebraic olympiad problems (Part I). Periodical scientific journal ,,Intelekti", №3(59), Tbilisi, 2017, pp. 20-24.
7. Gokadze I., Berdzulishvili G., Bregadze G. Geometric interpretation of the algebraic olympiad problems (Part II). Periodical scientific journal ,,Intelekti", №3(59), Tbilisi, 2017, pp. 25-29.