Akaki Tsereteli STATE UNIVERSITY
Faculty of Exact and Natural Sciences

TAMAR KHACHIDZE

SOME PROBLEMS OF HIDDEN DYMANICAL SYMMETRIES IN RELATIVISTIC QUANTUM MECHANICS

AN ABSTRACT

From the presented dissertation for obtaing the academic degree of Doctor of Physical Science

Kutaisi
2010

The Dissertation is accomplished at the Akaki Tseretli State University (ATSU) Depaртament of Physics and Tbilisi State University (TSU) Depaртament of Physics

Supervisors: 1. Prof. **Anzor Khelashvili**
Doctor of Physical and Mathematical Sciences,
Deputy Member of Georgian Academy of Sciences

2. Prof. **Davit Nishnianidze**
Full Professor of Akaki Tseretli State University

Reviewers: 1. Mr. **Alexsander Kvinikhidze**
Doctor of Physical and Mathematical Sciences,
Deputy Member of Georgian Academy of Sciences

2. Mr. **Gela Devidze**
Doctor of Physical and Mathematical Sciences,
Major Scientist at the High Energy Pysics Inst. TSU

The Defence of the Dissertation will be held on 25 december 2010, at 15:00 at the meeting of dissertation board of the Faculty of Exact and Natural Sciences Akaki Tseretli State University
Address: Block I, room 1114, 59 Tamar Mepe St., Kutaisi 4600.

The Dissertation will be available in the Akaki Tseretli State University Library (59 Tamar Mepe St., Kutaisi, 4600)
The Abstract of the Disertacion is dispached on 29.11.10.

The Secretary of the Dissertation Council *Zaza Sokhadze*

Subject of Research Work

The aim of this research is to study the hidden dynamical symmetries of the Kepler-Coulomb problem in classical and quantum mechanics non-relativistic and relativistic. Our main task is derivation of the symmetry genarators and to extract the physical consequences from their algebra. Consruct the superalgebra N=2.

Actuality of Problem

It is well known that symmetries, conservation laws and degeneracy of motion in physics are interrelated strongly. According to famous Nother's theorem invariance of system under some continious transformations entails the existence of corresponding integrals of motion, wich simultaneously play a role of generators of appropriate infinitesimal transformations. Invariance, that arises in this manner, as a rule, has **a geometric nature** - it descibes the degeneracy of motion, connected to coordinate transformations.

There are also symmetries which differ by its nature from the geometric ones. They take place only for particular kind of interactions (dynamics). They are named as **dynamical symmetries**. By this name it is underlined that they follow from spectial form of equations of motion only for certain forces (potentials).

In physics (Classical, as well in Quantum Mechanics) two forces are distinguished – **Coulombic and isotropic elasticity forces**. According to the Bertrand theorem only for them are allowed the periodic motion on closed orbits, and it follows the "accidental" degeneracy in Quantum Mechanics. The nature of this symmetry was explained by V.Fock, V.Bargman, W. Pauli and others. The corresponding conserved quantity is the **Laplace-Runge-Lenz (LRL) vector**.

This symmetry is spoiled for relativistic mechanics, but some traces of it still remainds, because the closed orbits occur in special rotated coordinate frame (Sommerfeld).

In this respect it is very importatnt to clear up the validity of hidden symmetry in relativistic quantum mechanics – particularly, in the Dirac equation. In 50-ies

Johnson and Lippmann introduced a pseudoscaler operator, which commutes with the Dirac Hamiltonian in the Coulomb field (Dirac-Coulomb Problem) . After, in 60-ies the Johnson- Lippmann (JL) operator was used many times for best separation of variables in the Dirac equation and for obtaining more simpliest ways of its solution. This problem becomes espacially actuall after the appearance of supersymmetric quantum mechanics in the 80's.

In spite of much interest generally to this problem and to the JL operator, particularly, there wasn't any publications in which the JL operator will be deduced regularly untill our work (2005). Motivated by the fact that in existed literature the derivation of the JL operator is omitted because many authors indicate that to show its commutativity with Hamiltonian requires quite tedious calculations it is desirable to find a simple and transparent method for construction of such an operator.

Probably for this reason the JL operator does not appear in textbooks on relativistic quantum mechanics, with only exception of a short footnote included in the book by B.Berestetskii et al.

We think that the importance of the JL operator will grow for further study of dynamical symmetries because of its connection to the LRL vector and "accidental" degeneracy.

By our opinion it is necessary to develop a regular procedure for deriving of JL operator, and moreover to clear up wether the Dirac equation has or not some extra symmetry for potentials other than Coulombic.

The result is generalized for any general combination of Lorentz-skalar and 1^{th} component of a Lorentz-vector.

It follows that the actuality of this problem is undoubtful.

Scientific Novelty

The general method of derivation of hidden symmetry operator for the Dirac equation is developed. This method is based on the description of degeneracy of Dirac-Coulomb spectrum with respect to two signs of κ, where κ is the eigenvalue of Dirac's K operator. Our method consists of two steps: First, we prove the theorem about the most general form of anticommuting with K operators (called K-odd operators). Then, we find such combination of K- odd operators, that commutes with the Dirac Hamiltonian, H. So, obtained operator describes tha additional symmetry of the Dirac Hamiltonian. This is a new and original result of the author, and is published in the scientific joural for the first time. The author's approach guarantees not only derivation of JL operator, but simultaneously its commutativity with the Dirac Hamiltonian is proved. It is very simple and transparent method.

When this method was aplied to the case of arbitrary central potentials, it is proved that the additional symmetry in the Dirac equation appears only for Coulombic potential. By this result the equivalence (ballance) to the classical mechanics is restored.

We construct N=2 superalgebra, include the Lorentz-scalar potencial, and derive the spectrum of the Dirac equation for general Coulomb potential (vector and scalar combination) algebraically, without consideration or solution of radial differential equation of motion.

It is proved a theoretical ground of the origin of Coulomb potential.

All of these results are derived and published by author for the first time.

Practicl Value

As we have mentioned above, the method of constuction of the symmetry operator developed by us, is general. Therefore it is applicable for study of similar theoretical problems. Also it may be possible to derive the spectrum of the combine systems algebraically without solution of equation of motion.

It is doubtless, that the results, derived by the author will be used in textbooks on relativistic quantum mechanics., especially after its publication in **American Journal of Physics,** as the paper dedicated to the problems of education at universities.

It must be mentioned our monograph:"Dynamical Symmetry of the Kepler-coulomb problem in Classical and Quantum mechanics: Non-Relativistic and relativistic", wich was published in New York by **Nova Science Publishers.** This monograph summarizes the major developments that have taken place in the problem of hidden or dynamical symmetries in classical and quantum mechanics (both non-relativistic and relativistic).

It is remarkable also to note that students of physics department of TSU already used these materials at their Seminars. These methods will take root in textbooks on Relativistic Quantum Mechanics and Quantum Electrodynamics. Let's mention also that our method may be extended to the cases of arbitrary spin particles practically without essential changes.

Main Results Presented for Defence

The Following Results derived in our Dissertation, are presented for the Defence:

1. The Theorem about K -odd operators, wich gives us the possibility to construct the operator for additional symmetries , and construct of Witten's SUSY algebra - N=2 SUSY.
2. Derivation of the operator for additional symmetry in case of Coulomb potential in the Dirac equation and demonstration its commutativity with the Dirac Hamiltonian.
3. Comment about Lemb Shift $nS_{1/2} - nP_{1/2}$, as a residual symmetry, wich is controlled by obtained additional symmetry.
4. The additional symmetry of the Dirac Hamiltonian for Coulomb field is **supersymmetry,** which deeply is manifestation of symmetry connected to the LRL vector.
5. General Proof about that the above considered symmetry as degeneracy , connected to two signs of eigenvalues of K -operator , is the property of **Coulombic potential** only.
6. Comments about symmetries in the squared Dirac equation. It appears, that we must make some caution when the Dirac equation is solved after its squaring, because the inner nature of equation may be changed particularly when infinitely rising potentials are considered. This comment is concerned with harmonic oscillator.
7. Jonson-Lippman operator is generalized when in Hamiltonian with a 4^{th} component of a Lorentz-vector we have a Lorentz-scalar potencial.
8. Algebraic derivation of the spectrum of the Dirac Hamiltonian for an arbitrary combination of the Lorentz-scalar and Lorentz-vector Coulomb potential.
9. Theoretical ground of the Coulomb potencial.

Structure and the basic Contents of the Thesis

The thesis consists of Introduction, four Chapters, two Appendices and the List of References with 55 items. In summary all of this takes 88 pages. There are also four figures.

Introduction

In the introductory part the main subjects of the Thesis are presented and the short overview of our task is done. There are described relation between symmetries, conservation laws and the degeneracy of motion. The differences between geometric and dynamical symmetries are explained.

As it appears the dynamical symmetry of Coulombic problem is practically well studied in classical and quantum mechanics non-relativistic and relativistic. At the same time the problems solved in this Thesis are overviewed.

Chapter I

In this chapter the so called "hidden" symmetry in classical mechanics is reviewed. The closeness and strict periodicity of planetary orbits the modern physics ascribes to the existence of extra conservation laws, which takes place only for Coulomb potential. It is **the Laplace-Runge-Lenz vector.**

Initial part of this chapter is devoted to the derivation of this vector from the Newton's equation of motion in the central potential. After certain manipulations it appears that the following vector

$$\vec{A} = \left[\vec{p} \times \vec{l}\,\right] - ma\hat{r} \qquad (1)$$

is conserved. Here $\hat{r}$ is the unit vector in the direction of radius-vector, and a is the strength of Coulombic force

$$V_C(r) = -\frac{a}{r}, \qquad (a>0) \qquad (2)$$

Among applications of this vector there is in Thesis the derivation of orbit equation algebraically, i.e. without solving of equation of motion. Algebraic aspects are also considered by using the Poisson brackets. It is shown that this vector together with the orbital momentum vector $\vec{l}$ forms closed algerbraic relations under the Poisson brackets, which is isomorphic to the algebra of $SO(4)$ group of rotations in 4-dimensional Euclidean space. Therefore to finite motion in classical mechanics we have a larger group of symmetry, than ordinary 3-rotations. By this reason the additional conservation law is called as a "hidden" symmetry.

Some aspects of the extended symmetry are considered as well. Particularly, we overview one very omportant, by our opinion, paper [1], in which the post-newtonian approximation is studied and the relation between the LRL vector and the Lorentz boost vector is established. According to this relation the conservation of one of them is a consequence of conservation of another vector. It means that for the conservaton of LRL vector dynamics must follow to relativistic Poincare group, which is very strong requirement.

Chapter II.

In this chapter the role of LRL vector in NRQM is disscussed. The first, who gave attention to this problem, was V.Pauli [2] in 1926, then appeared papers by V.Fock [3], V.Bargman [4] and others, which made very important contributions. In pass to quantum mechanics we have to change

$$\vec{A} = \hat{r} - \frac{1}{2ma}\left[\vec{p} \times \vec{l} - \vec{l} \times \vec{p}\right] \qquad (3)$$

and introduce corresponding operators. This quantum operator commutes with the Schroedinger Hamiltonian

$$H_{NR} = \frac{p^2}{2m} - \frac{a}{r} \qquad (4)$$

and together with the momentum operator $\vec{l}$ forms algebra isomorphic to $SO(4)$.

There is described how the extednded algebras appear in connection to signs of total energy. In the case of negative energy, when we must have bound states, the well known Balmer's formula for hydrogen atom spectrum is derived by algebraic methods only.

Here we disscussed also papers, concerned to supersymmetry of coulomb problem in NRQM. This is considered on the example of Pauli electron. It appears that such a problem has supersymmetry, where the role of supercharge plays the projection of LRL vector on the electron's spin direction.

Chapter III.

Following two chapters are devoted to the original results, derived by author. The additional symmetry is studied in relativistic quantum mechanics, particularly, in the Dirac equation. The strategy follows to the line – **from Pauli to Dirac**, which means the following: Started point is the Dirac operator

$$K = \beta(\vec{\Sigma} \cdot \vec{l} + 1), \qquad (5)$$

which commutes with the Dirac Hamiltonian in arbitrary central symmetry potential

$$H = \vec{\alpha} \cdot \vec{p} + \beta m + V(r), \quad [K, H] = 0 \qquad (6)$$

This symmetry is well known, but its properties are less studied.

Because of this symmetry the spectrum of Dirac equation has an additional degeneracy, connected to the two signs of the eigenvalues κ of the Dirac K operator.

We must introduce an operator, which mixes these signs, it is evident, that such an operator must anticommute with K. Such an operator was introduced in 50-ies by Johnson and Lippmann [5]. But they had published only the short abstract. As regards of Detaled derivation of this operator, it is not published in scientific literature up to now- one of the curious case in physics history. This operator has the following form (in modern notations) [6]:

$$A = \gamma_5 \vec{\alpha} \cdot \hat{r} - \frac{i}{ma} K \gamma_5 (H - \beta m) \qquad (7)$$

It is worthwhile to note that this operator was often used in scientific literature: in 60-ies - to find easier ways for the solution of Dirac-Coulomb problem, [7] while in 80-ies the interest to it was renewed from SUSY point of view [8] . It is interesting that recently [9] the generalization of this operator for general dimensions was considered. but authors didn't pay attention to the derivation of this operator and it is noted only, that to show commutativity of this operator with the Dirac Hamiltonian is a "very longtime and tadious task".

The proof of this proerty is described in the Appendix I in our thesis. Because of anticommutativity

$$\{A, K\} = 0 \qquad (8)$$

it is elementary to construct the Witten's algebra (which correspondes to $N = 2$ supersymmetry). For this aim the supercharge operators must be chosen as follows [10]

$$Q_1 = A, \qquad Q_2 = i\frac{AK}{\kappa} \qquad (9)$$

It is evident that

$$\{Q_1, Q_2\} = 0 \text{ and } Q_1^2 = Q_2^2 = A^2 \equiv h \qquad (10)$$

Hence the Witten's algebra is deduced, where h is a Witten's Hamiltonian. It differs from the Dirac Hamiltonian.

Calculations, performed in Appendix II give the relation

$$A^2 = 1 + \left(\frac{K}{a}\right)^2 \left(\frac{H^2}{m^2} - 1\right) \qquad (11)$$

We see, that the Witten's Hamiltonian is expressed by the square of Dirac Hamiltonian, in which all enries commute with each others. Therefore passing to their eigenvalues, we obtain the energy expression without solving of equation of motion

$$E^2 = m^2 \left\{ 1 + \left(\frac{Z\alpha}{\kappa}\right)^2 \left(A^2 - 1\right) \right\} \qquad (12)$$

This relation gives a possibility by using Witten's algebra to build the ladder procedure, introducing nilpotent operators $Q_{\pm} = Q_1 \pm iQ_2$ and derive the famous Sommerfeld formula for hydrogen spectrum $[11]$:

$$E_n = m \left\{ 1 + \frac{(Z\alpha)^2}{\left(n - |\kappa| + \sqrt{\kappa^2 - (Z\alpha)^2}\right)^2} \right\}^{-1/2} \qquad (13)$$

From the group theoretical point of view this result means the following: Nonrelativistic hydrogen atom had a symmetry, connected to the conservation of the LRL vector. Hence the symmetry enlarged till $SO(4)$, instead of space rotation symmetry $SO(3)$. In passing to relativistic case this symmety is lowering partly and becomes $SO(3) \times S(2)$, due to N=2 supersymmetry. It is very interesting, the supersymmetry appears because of hidden symmetry, wich now is connected to JL operator. In Figure below the degeneracy of hydrogen levels with respect to two signs of κ is displayed clearly

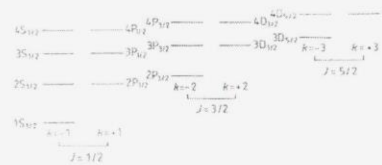

Figure Relativistic Spectrum of Hydrogen Atom, characteristic for $S(2)$ *supersymmetry*

As it is known the real spectrum shows the level shift $nS_{1/2} - nP_{1/2}$, so called Lamb Shift. This shift is explained only in the framework of QED. the Lamb Shift Hamiltonian at the one loop order has the form [12]

$$\Delta V_{eff} \,\square\, \frac{4\alpha^2}{3m^2}\left(\ln\frac{m}{\mu} - \frac{1}{5}\right)\delta(\vec{r}) + \frac{\alpha^2}{2\pi m^2 r^3}\left(\vec{\Sigma}\cdot\vec{l}\right) \qquad (14)$$

It is easy to convince that the JL operator doesn't commute with these terms. So the inclusion the Lamb-shift terms into the Dirac Hamiltonian breaks the above symmetry. This gives us possibility to suggest that the breakdown of symmetry, connected to the JL operator , causes the appearance of Lamb Shift in the Dirac Hamiltonian.

Next point is the theory of the JL operator. First of all we prove the following theorem :

Theorem: If $\vec{V}$ is a vector with respect to angular momentum $\vec{l}$ and simultaneously is perpendicular to it $\vec{l}\cdot\vec{V} = \vec{V}\cdot\vec{l} = 0$, then the operator $\left(\vec{\Sigma}\cdot\vec{V}\right)$ anticommutes with K: $\left(\vec{\Sigma}\cdot\vec{V}\right)K = -K\left(\vec{\Sigma}\cdot\vec{V}\right)$ (15)

The proof of this theorem is rather easy and is published in our paper [13].

Now we are able to construct the general class of the K-odd operators and find the one that commutes with the Dirac Hamiltonian We derive that the operator with required properties is [10];

$$A' = \vec{\Sigma}\cdot\vec{n} - \frac{i}{ma}K\left(\vec{\Sigma}\cdot\vec{p}\right) + \frac{i}{mr}K\gamma_5 \qquad (16)$$

It is K-odd, in accord of our theorem and commutes with the Dirac Hamiltonian.

Hence, we have derived desired symmetry operator and in parallel demonstrate its commutativity with the Dirac-Coulomb Hamiltonian. It is evident, that the proof of these properties appears to be very simple and transparent.

After that we made use the obvious properties of Dirac matrices and show that this operator coincides with JL operator, Eq.(7).

Chapter IV

As our procedure is rather simple and powerful, there arise natural question – How to study the problem of additional symmetry in arbitrary central potential fields, especially as the Dirac's K operator commutes with the Hamiltonian in any case.

We have used our theorem and construct the most general K-odd operator, that can be build from the physical vectors at hand, namely $\vec{n}$ and $\vec{p}$, which obeys the conditions of theorem,

$$A = x_1\left(\vec{\Sigma}\cdot\vec{n}\right) + ix_2K\left(\vec{\Sigma}\cdot\vec{p}\right) + ix_3K\gamma_5 f(r) \qquad (17)$$

After calculating its commutator with the Dirac Hamiltonian and requiring the vanishing of result, we are faced with the following equations

$$x_2V'(r) = x_3f'(r), \quad x_3mf(r) = \frac{x_1}{r} \qquad (18)$$

It follows that

$$V(r) = \frac{x_1}{x_2}\frac{i}{mr} \qquad (19)$$

Therefore under very general conditions, it follows that:

- **only central potential, for which the Dirac Hamiltonian has an additional symmetry (in the abovementioned sense) is the Coulomb potential.** Moreover the symmetry takes place both for attraction and repulsion.

In case of Coulomb potential our operator becomes the ordinary JL one.

We note at the same time, that while the K - symmetry remains when potential is Lorenz-scalar, i.e. $V \to \beta V$, but in this case obtained symmetry operator (17) does not commute with the Hamiltonian.

Therefore we are convinced that in the problem of additional symmetry the Coulomb potential takes a peculiar place – only for it have we Witten's superalgebra, which simultaneously is the symmetry of Dirac's Hamiltonian.

After that we are interested in question – what the real physical picture is standing behind this? Accordig the derived relations our operator may be reduced to the following form:

$$A = \vec{\Sigma}\cdot\left(\vec{n} - \frac{i}{2ma}\beta\left[\vec{p}\times\vec{l} - \vec{l}\times\vec{p}\right]\right) + \frac{i}{mr}K\gamma_5 \qquad (20)$$

which in nonrelativistic limit $\beta \to 1$, $\gamma_5 \to 0$ reduces to the following expression

$$A \to A_{NR} = \vec{\sigma}\cdot\left(\vec{n} - \frac{i}{2ma}\left[\vec{p}\times\vec{l} - \vec{l}\times\vec{p}\right]\right) \qquad (21)$$

Expression in parenthesis is the LRL vector, Therefore in NR limit the relativistic supercharge A is reduced to the projection of LRL vector on the electron's spin direction. Precisely this operator was used in the case of Pauli electron.

Therefore we can conclude that the symmetry connected to the LRL vector includes very wide range of physical phenomena from planetary motion to fine and hyperfine structures of atoms.

It is noticeable that the hidden symmetry, familiar to Coulomb potential, manifests here itself unexpeditely only at the end of calculations.

At the last point of this thesis we discuss the general comments about the dynamical symmetries in Dirac equation. We remember that according to the Bertrand theorem in classical mechanics there is another potential – isotropic harmonic potential – for which orbits are closed as well. Where is such potential in Dirac's case? We have shown that for the considered form of Dirac Hamiltonian, Eq.(6), for arbitrary infinitely rising potentials, the well known Klein paradox [14] takes place and solutions, corresponding to bound states asymptotics of the wave function, disappear[16].

We consider two examples with vectorlike potentials , which are switched on by minimal anzatz, $\vec{p} \to \vec{p} - \vec{A}$, and show that in two cases , namely $\vec{A} = B[\vec{n} \times \vec{r}]$ - motion in uniform magnetic field and $\vec{A} = im\omega\beta\vec{r}$ - so called, "Dirac oscillator" the Dirac equation has proper asymptotics for bound states. In is remarked also that squaring of Dirac equation may change the inner nature of the Equation and the certain caution is necessary when infinitelly rising potentials are into consideration.

We include the Lorentz – scalar potencial as well:

$$H = \vec{\alpha} \cdot \vec{p} + \beta m + V(r) + \beta S(r) \qquad (1)$$

This Hamiltonian commutes wit K -operator again, but *does not commute* with the above derived JL operator. It is clear from physical reason that the supercharge pperator has to be generalized to this case. For this purpose one must increase number of K-odd structures, making use of above mentioned theorem. Let us probe the following operator:

$$\mathrm{X} = x_1(\vec{\Sigma} \cdot \vec{r}) + x_1'(\vec{\Sigma} \cdot \vec{r})H + ix_2K(\vec{\Sigma} \cdot \vec{p}) + ix_3K\gamma_5 f_1(r) + ix_3'K\gamma_5\beta f_2(r) \quad (2)$$

Here we included for ϑ the Hamiltonian H in the first structure and at the same time the matrix β in the third structure. Both of them commute with K . The form (2) is a minimal extension of the previous operator, because only the first order structures in $\vec{r}$ and $\vec{p}$ participate. Calculation of relevant commutators with the Hamiltonian (1) and requirement of symmetry gives the consistency equations, from which it follows that both potentials must be Coulombic: $S(r) = \dfrac{x_1'}{x_2 r}$

$$V(r) = \frac{x_1}{x_2 m r} \qquad (3)$$

Therefore we make sure that N=2 supersymmetry in the above context is a symmetry of the Dirac Hamiltonian only for Coulomb potential (for any general combination of Lorentz – scalar and 4^{th} component of a Lorentz- vector).

From the above obtained solutions one can reduce the new supercharge operator to more compact form [18]:

$$\mathrm{X} = (\vec{\Sigma} \cdot \vec{r})(ma_V + Ha_S) - iK\gamma_5(H - \beta m) \qquad (4)$$

Now we illustrate the advantage of our approach in deriving spectrum pure algebraically using Witten's superalgebra, established above. To explore this algebra, let define a SUSY ground state $|0\rangle$ as follows:

$$\widetilde{H}|0\rangle = \mathrm{X}^2|0\rangle = 0 \ \to X|0\rangle = 0 \qquad (5)$$

Bcause X^2 is a square of Hermitian operator, it has a positive definite spectrum and one is competent to take zero this operator itself in ground state. By thys requirement we'll obtain Hamiltonian in this ground state and, consequently ground state energy.

So let equate $X = 0$ and solve H. Then we diagonalize this expression by using Foldy- Wouthuysen like transformation[19]. We need at least two such transformacions [17]. Finally for the ground state energy we obtain:

$$H_0 = E_0 = \frac{m}{k^2 + a_S^2}\left[-a_S a_V \pm k\sqrt{k^2 - a_V^2 + a_S^2}\right] \qquad (6)$$

It coincides with the expression, derived by explicit solution of Dirac equacion [20]. For obptaining of total spectrum it is now to use the Witten's algebraic step procedure, which for our case consists in change $\gamma \to \gamma + n - |k|$, where $\gamma^2 = k^2 - a_V^2 + a_S^2$. Making use of this substitucion into (6), the correct expression for total energy spectrum follows[20].

The Main Results of the Thesis

Let's enumerate the main results, obtained in this Thesis :

a) The theorem about K-odd operators is generalized to relativistic quantum mechanics.

b) On the framework of this theorem the K - odd operator is constructed, which commutes with the Dirac Hamiltonian in case of Coulomb potential. It descibes the hidden symmetry of Hamiltonian and coincides to the JL operator.

Derivation of this operator is **author's original result**, the first publication belongs to author.

c) It is shown that this operator and symmetry it generates is a generalization of known from classical mechanics LRL vector's symmetry to relativistic case. At the same time this symmetry controls the Lamb Shift in the hydrogen atom spectrum.

d) The developed in Thesis theory is applied to the Dirac Hamiltonian for arbitrary central potential and it is proved that the symmetry takes place only for Coulomb potential.

e) It is cleared up that the hidden symmetry of Coulomb problem is N=2 supersymmetry.

f) It is proved that due to the Klein paradox there appear no additional symmetry for oscillator potential in the Dirac Hamiltonian . It is absent for any infinitelly rising central potentials.

g) It is shown the **unique role of the Coulomb potencial** for any general combination of Lorentz-scalar and 4^{th} component of a Lorentz-vector - both potencial must be Columbic.

h) Spectrum of the Dirac Equation is obtained algebraically for arbitrary combination of Lorentz-scalar and 4^{th} component of a Lorentz-vector without consideration or solution of radial differential equation of motion.

i) It is proved that the Witten's superalgebra based on Dirac's K-operator and anticommuting with it Johnson-Lippman like operator may serve as a theoretical ground of the origin of Coulomb potential.

Approbation of the Work

This thesis is accomplished at the Theoretical Physics Department of Tbilisi State University and disscused several times at the Mamasakhlisov's Seminars.

Parts of Thesis were reported at the several Conferences, such as: "UNESKO" Seminar in Theoretical Physics (Tbilisi, IX,2005),

Yang Physicists' Conference, dedicated to A.Einstein's famous papers (Tbilisi, XI,2005), International Conference on High Energy Physics, CICHEP II, Cairo, january, 2006.

Talk at the School and Conference-**"New Trends on High Energy physics"**, Crimea, Yalta, 16-23 sept. 2006.

International Conference **SQS-07, Dubna, 2008.**

REFERENCES:

1. **Dahl J.D.**, Journ.Phys.A., Math. Gen., **30,** 6831-6890);
2. **Pauli W.**, Z.f.Fizik.,**36**, 336-363 (1926);
3. **Fock V. A.**, Z. f. Fizik., **98**, 145-154 (1935);
4. **Bargman V.**, Z.f.Fizik., **99**, 168-188 (1936);
5. **Johnson M.H. and Lippmann B.A.**, Phys.Rev., **78,**229(A), (1950);
6. **Stahlhofen A.A.**, Helv. Phys. Acta, **70,** 372-386 (1997);
7. **Biedenharn L.C.**, Phys. Rev., **126**, 845-851 (1962);
8. **Sukumar C.V.**, J.Phys.A: Math. Gen., **18**, L697-701 (1985);
9. **Dahl J.P. and Jorgensen T.**, Int.J. of Quantum Chemistry, **53**, 161-181 (1995);
10. **Katsura H. and Aoki H.**, Journ. Math. Physics, **47**, 032301 (2006);
11. **Khachidze T.T. and Khelashvili A.A.**, Modern Phys. Letters, **A20**, 2277-2281 (2005);

12. **Sommerfeld A.**, "atombau und spectrum", (Vieweg, Braunschweig, 1929);
13. **Kaku M.**, Quantum Field Theory., (Oxford Univ. Press, 1993);
14. **Khachidze T.T. and Khelashvili A.A.**, Am. J. Physics, July-August (2006);
15. **Klein O.**, Z.f.Fizik, **53**,157 (1929);
16. **Khachidze T.T. and Nadareishvili T.P., Bull. Of Georg. Acad.Sci., 174,** 61-64 (2006);
17. **Khachidze T.T. and Khelashvili A.A., Ukr. Fiz. Zhurnal (UFZ), 52 , N5** 421-423 (2007);
18. **Leviatan A.** Phys.Rev. lett. **92,** 202501 (2004);
19. **Foldi L. and Wouthuysen S.** Phys.Rev. 72, 29 (1950);
20. **Greiner W., Muler B. and Rafelski J.** Quantum Eleqtrodynamics of Strong Fields. Springer-Verlag, (1985).
21. **Witten E.** Nucl. Phys. B188, 513 (1981);

The Results of this **THESIS** are based on the following **PUBLICATIONS:**

1. **T.T. Khachidze and A.A. Khelashvili**, "An "Accidental" Symmetry Operator for the Dirac Equation in the Coulomb Potential", **Modern Phys. Letters, A20, 2277-2281 (2005)** And ArXiv: **hep-th/0507247.**
2. **T.T. Khachidze and A.A. Khelashvili**, "Manifestations the Hidden Symmetry of Coulomb Problem in the Relativistic Quantum mechanics - from Pauli toDirac Electron", **Bull.of Georg. Acad. Sci., 172,452- 455 (2005)**; and ArXiv: **quant-ph/0507257.**
3. **T.T. Khachidze and A.A. Khelashvili**, "The Hidden Simmetry Operator of the Kepler Problem in Relativistic Quantum Mechanics – from pauli to Dirac," **American Journ. Of Physic, 74, 628-632 (2006)** and ArXiv: **physics-ph/0508122.**
4. **T.T. Khachidze and A.A. Khelashvili**, "Supercharge Operator of Hidden Symmetry in the Dirac Equation", Talk at the Int. Conf. on High Energy Physics, **CICHEP II**, Cairo, jan. 2006 (see, Proceedings **Cairo Int. Conference on High Energy Physics, CICHEP II), 279-984 (2007)** and ArXiv: **hep-th/0602181.** დაიბეჭდა აგრეთვე თსუ ჟურნალში **"Physics",2006, v.40,97-112.**
5. თ. ხაჩიძე, "კულონური ურთიერთქმედების "ფარული" სიმეტრიის გამოვლინებანი რელატივისტურ კვანტურ მექანიკაში", კრებულში "და იყო დღე ფიზიკისა", გვ. 138–145, თბილისი, 2005.
6. **T.T. Khachidze and T. P. Nadareishvili,** "Dirac Equation and its Squaered Form", **Bull. Of Georg. Acad.Sci., 174,** 61-64 (2006), and ArXiv: **hep-th/0606043.**
7. **T.T. Khachidze and A.A. Khelashvili,** "On the Nature of Hidden Symmetry or accidental degeneracy of the Kepler Problem". Talk at the School and Conference-**"New Trends on High Energy physics"**, Crimea, Yalta, 16-23 sept. 2006. **in Proceedings of this Conference**. And Arxiv: **hep-th/0610139.**

8. **T.T. Khachidze and A.A. Khelashvili,** "Algebraic Derevation of Spectrum of the Dirac Equation for Arbitrery Combination of Lorentz-scalar and Lorentz – vector Coulomb potentials", **Ukr. Fiz. Zhurnal (UFZ), 52 , N5** 421-423 (2007) and ArXiv: **hep-th/0612199**.
9. **T.T. Khachidze and A.A. Khelashvili,** "On the quantum Relativistic Origin of the Coulomb Potential", **Bull. Of Georg. Acad. of Sciences, Vol. 1(175), N4.**
10. **T.T. Khachidze and A.A. Khelashvili,** "Supersymmetry in the Dirac Equation for Generalized Coulomb Potential", Arxiv: **hep-th/0701259 (2007).**
11. **T.T. Khachidze and A.A. Khelashvili,** "N=2 Supersymmetry in the Dirac equation – Possible motivation for Coulomb potential", **Proc. SQS-07, Dubna, 2008**.
12. **T.T. Khachidze and A.A. Khelashvili,** " Dynamical symmetries of the Kepler – Koulomb Problem in classical and Quantum Mechanics (Non-relativistic and Relativistic), **Monograph. Nova Publishers, New York, 2008.**
13. **T.T. Khachidze and A.A. Khelashvili,** "Why the Coulomb Potential?", **Applied and Computational Mathematics, An International Journal (in press, 2009)**